

Chapter 1

Introduction

This chapter provides an overview of network controllability and controllability robustness, presenting the development from classical control theory to modern network science. It reviews some foundational concepts, including structural and exact controllability, highlights recent advances addressing conventional, temporal and multilayer networks, and discusses energy-efficient and targeted control strategies. It examines the growing importance of robustness against random failures and hostile attacks, and formulates key research questions on maintaining functionality and optimizing robustness. It outlines the emerging machine learning techniques for analyzing dynamics, evaluating robustness, and optimizing structures. The chapter concludes with the motivation and organization of the book, positioning controllability robustness as a critical challenge for secure and resilient networked systems in dynamic and adversarial environments.

The chapter is organized as follows. Section 1.1 presents a brief overview of network controllability and controllability robustness. Section 1.2 discusses practical applications of the subjects across diverse domains. Section 1.3 examines various risks posed by adversarial attacks and their implications for network controllability robustness. Section 1.4 provides the motivation and outlines the structure of the book.

1.1 Overview of Network Controllability and Controllability Robustness

Controllability is a fundamental concept in control theory and network science, indicating the ability to steer a networked system from an initial state to a target state by a limited external input, called control, in finite time.

In this book, only networks of linear or linearized dynamical systems are studied. Recently, research on such linear network controllability has attracted substantial attention because of the significance and applicability of this concept across diverse domains, including biological networks such as gene regulatory and metabolic pathways, neural networks underlying cognitive processes, epidemiological networks modeling disease transmission, social networks governing information diffusion and influence propagation, engineered infrastructures such as power grids and transportation systems, logistical networks ensuring supply chain robustness, financial networks managing systemic risks, and communication networks supporting large-scale data transmission and distributed computing [1–5].

Verifying the controllability of a linear or linearized dynamical network is vital in engineering and technology, which ensures that a system in concern can meet theoretical and practical requirements for designated control tasks as well as reliability under varying conditions. Common methods include the Kalman rank condition [1, 6–8], the Popov-Belevitch-Hautus (PBH) rank condition [9], and the graph-theoretic criterion for structural controllability [10]. Simulation-based and energy-based approaches [11] are also useful for evaluating network controllability under realistic constraints, considering essential issues such as network structures, control nodes, and component dynamics.

The origins of network controllability can be traced back to the classical control theory on linear dynamical systems, particularly the foundational work of Kalman in the 1960s [1, 6, 8]. Kalman introduced the concept of controllability and established a criterion based on a system matrix, called the controllability matrix, which provides a rigorous mathematical framework for determining whether a linear system can be fully controlled by bounded external inputs in finite time. This principle was later extended to interconnected linear dynamical systems, forming a basis for the network control theory applicable to complex dynamical networks.

The classical notion of controllability in control theory primarily addresses the intrinsic dynamics of a single higher-dimensional linear system. With the emergence of network science in the early 2000s, one research focus was the extension of the system controllability toward the controllability of large-scale complex networks [12–14]. In such networks, nodes represent dynamical subsystems and edges capture the interactions among them, encompassing scientific, engineering and technological domains such as the Internet, power grids, transportation systems, neural networks, social networks and financial networks [4].

In the era of pervasive connectivity, Liu, Slotine, and Barabási advanced the classical concept of structural controllability [10], establishing a fundamental link between directed network topology and system controllability by introducing a driver node algorithm based on the maximum matching principle from graph theory, which identifies the minimal set of nodes required to achieve full control of the network.

Building on this notion, the framework of exact controllability emerged as a complementary approach to refine the understanding of network controllability [9]. It determines the network controllability by considering both network topology and precise numerical values of system parameters. Differing from the structural controllability, which depends solely on the zero or nonzero structure of the network controllability matrix and holds for almost all parameter choices, exact controllability provides a parameter-specific analysis. The minimal number of driver nodes required for full control of the network is equal to the maximum geometric multiplicity of the eigenvalues of the system controllability matrix, or the maximum algebraic multiplicity for diagonalizable undirected networks. Driver node locations are identified using the PBH rank condition with appropriate elementary matrix transformations. This approach applies to networks with arbitrary structures and edge weights and provides a universal tool for assessing controllability of directed or undirected complex networks.

Recent advances in network controllability extend beyond foundational state and structural controllability, addressing increasingly complex challenges. Key research directions include nonlinear networks [15], where progress in Lyapunov methods and basin-of-attraction analyses has enhanced stability control and feasibility for complex systems such as gene regulatory and ecological networks. Temporal and time-varying networks, with evolving connections in transportation and communication systems, require time-varying dynamic frameworks [16] and higher-order models to capture the impact of time-dependent interactions on the network controllability [17]. Energy-efficient control, critical for large-scale or resource-constrained networks [18], focuses on optimizing the number and location of driver nodes, reducing control energy, and ensuring robustness using fault-tolerant strategies for infrastructures such as power grids and cyber-physical networks. In addition, new directions include targeted control of specific nodes [19] and control of multi-relational and multilayer networks [20], all emphasizing practical applications and experimental verification to ensure scalability and real-world relevance.

Among the above key topics, a particular important and fundamental issue in network controllability is *robustness*, which raises two central research questions to be addressed by this book:

- *How can complex networks maintain intended functionality under structural failures and attacks?*
- *How can this capability be evaluated, analyzed and optimized?*

Structural disruptions can result from random failures such as natural disaster events, aging components, and unforeseen breakdowns, as well as from deliberate malicious attacks including cyber destruction and sabotage, often targeting critical nodes or edges to maximize the effect and impact. Ensuring robustness is thus essential for reliable operations of critical systems such as power grids, communication networks, and transportation infrastructure under uncertain and adversarial conditions.

Recently, on the other hand, machine learning has become a powerful tool for advancing research on network controllability and robustness. Leveraging machine learning algorithms enables efficient analysis of complex system dynamics, identification of critical nodes, and optimization of control strategies. Machine learning supports behavior modeling, pattern discovery, adaptive control, and higher-dimensional network analysis, accelerating solutions and improving controllability across diverse applications. Furthermore, both attack strategies and protective measures have evolved significantly through integration of network science theory and machine learning techniques [21–26]. Addressing these challenges is crucial for designing resilient and secure systems capable of maintaining functionality in dynamic and hostile environments.

1.2 Practical Applications of Controllability and Controllability Robustness

Network controllability has wide-ranging applications. In neuroscience, controllability aids in understanding brain activities and influencing regions for therapy [27]. In power systems, controllability supports the design of resilient grids for stability and fault recovery [28]. In transportation networks, controllability optimizes routes, traffic flow, and resource allocation for efficient operations [29]. Without controllability, a system may fail to respond effectively to control inputs, leading to degraded performance or instability.

Recent research has advanced network controllability across diverse topological types and configurations [4, 5]. In multiplex and multilayer

networks, controllability is influenced by interlayer coupling and timescale differences. Control is more efficient when applied to the faster layer, and a critical timescale threshold determines whether control is dominated by one layer or shared between layers [30]. In evolving networks, a general rule predicts changes in the number of driver nodes during network growth, helping maintain controllability as new nodes and edges are added [31]. Regarding input-output configurations, comparative studies show that MIMO (multiple input multiple output) systems significantly outperform SISO (single input single output), SIMO (single input multiple output), and MISO (multiple input single output) systems in terms of channel capacity and reliability, making them ideal for high-performance communication and control systems [14, 32, 33]. While these improvements enhance control efficiency of a network, the structural controllability remains primarily determined by its topology.

Investigating network controllability focuses on analyzing the ability of a networked system, for those both already exist in the real world or are intended for future deployment, to be effectively fully controlled, while investigating controllability robustness extends this analysis by assessing the ability of such controllability under non-ideal conditions and in adversarial environments, ensuring its sustainability and maintenance.

Controllability robustness is essential for guaranteeing stable and continuous functionality, enabling fault recovery, and enhancing resilience, as it characterizes the capacity of a system to preserve the controllability under structural modifications or adverse conditions. Evaluating the controllability robustness of a system before real-world deployment is essential. Under stress tests conducted in intentionally designed non-ideal environments, where failures and attacks occur sequentially, system performance is measured in terms of the remaining controllability. It is then aggregated into an overall metric of controllability robustness, typically by averaging, although alternative computation methods are possible.

1.3 Adversarial Attacks and Controllability Robustness

Malicious attacks on complex networks have become increasingly critical in modern interconnected systems. Typical examples include Internet viruses, cyber intrusions, distributed denial of service (DDoS) attacks, and ransomware targeting critical infrastructure [34]. In addition, real-world systems encounter targeted disruptions on power grids and transportation networks, coordinated attacks on sensor networks, adversarial manipula-

tion on social and communication platforms, and structural degradation in biological or ecological environments. Such disruptions can propagate across interconnected infrastructures, amplifying systemic vulnerability.

Other attack forms include data corruption, which undermines the integrity of information processed by nodes [35]; communication interference, such as jamming in wireless networks [36]; overload attacks, where specific nodes or edges are intentionally stressed to induce systemic failures [37], and cascading failures, where an initial disruption propagates through the network, triggering a chain reaction of malfunctions [38]. Cyber attacks on power grids exemplify these risks, demonstrating the potential to cause large-scale blackouts and cascading crashes across interconnected facilities [39, 40]. The accelerating digitalization of power systems further enlarges the attack surface, heightening exposure to advanced persistent threats and coordinated intrusions [41]. Addressing these challenges from a general topological perspective is essential, while domain-specific expertise remains indispensable for safeguarding individual systems.

When real-world infrastructures are modeled as complex networks, their entities, such as power stations in power grids, routers and switches in communication systems, intersections and stations in transportation networks, or individuals in social networks, are represented as nodes (also called vertices), and their relationships, such as transmission lines, data links, traffic routes, or social interactions and acquaintanceship, are represented as edges (or links).

Malicious attacks, random failures, traffic congestion, dissolution of social relationships, and other environmental disturbances that alter network structures are commonly modeled by node removal or edge removal. In these scenarios, the focus extends beyond the physical nature of the damage to emphasize its structural and functional implications for the network.

Attacks on complex networks can be categorized along several dimensions. First, based on the target, they can be classified as node attacks, which aim to disable specific entities, and edge attacks, which seek to disrupt the connections among the entities. Second, according to the selection strategies, attacks may be random where nodes or edges are removed without preference, or targeted where the deemed most critical components are deliberately chosen to maximize the disruption. Third, depending on whether the importance of nodes or edges is updated during the attack process, attacks may be metric-reevaluated where the importance is recalculated after each removal, or metric-non-reevaluated where the initial ranking remains fixed. Finally, based on the methods used to assess

importance, attacks may be categorized as statistics-based which rely on metrics such as degree or betweenness, computational intelligence-based which employ heuristic algorithms such as evolutionary computation or swarm intelligence, and machine learning-based which leverage models such as graph neural networks and deep learning to identify key components automatically [42].

From an adversarial perspective, the advancement of computational intelligence and machine learning techniques has rendered both attack design and identification of critical components in complex networks increasingly adaptive and sophisticated. These developments enable adversaries to formulate precise and effective strategies, while simultaneously providing defenders with advanced tools for vulnerability detection and for implementing protective measures with greater accuracy and efficiency. Similarly, from the perspective of safeguarding networked systems, these advancements can be leveraged to strengthen the robustness and resilience, improve fault recoveries, and ensure reliable operations subject to evolving threats.

1.4 Motivation and Outline of the Book

Nowadays, as real-world environments become increasingly complex and life-threatening, robustness in network controllability has emerged as an important research focal issue. Progressively, robustness studies encompassing theoretical, computational and data-driven tasks have reached out to cover many other network functionalities such as connectivity and communication, which have contributed to the development of network science and technology.

This book addresses the fundamental challenge of assessing and enhancing network controllability robustness and provides insights into related topics, including robustness of additional network functionalities, graph learning with emphasis on performance evaluation, attack and defense strategies targeting network functionalities, and modeling random processes of malicious attacks on networks, among others.

1.4.1 Motivation

The rapid expansion of interconnected systems across natural, engineering, social, and biological domains has profoundly transformed their complexity to be perceived and managed in real-world applications. These networks deliver significant benefits to societies in terms of efficiency and connectivity.

Among their many functionalities and advantages, controllability is particularly important, meanwhile these systems also introduce substantial vulnerabilities. Malicious attacks, cascading failures, and unpredictable disruptions underscore the urgent need for resilient and intelligent control strategies of complex networks.

At the same time, breakthroughs in computational intelligence, particularly in graph learning, a specialized branch of machine learning that processes data represented as graph structures, and data-driven modeling, have created unprecedented opportunities to analyze, understand and optimize complex networks. These technologies provide effective tools for network analysis and computation, enabling discovery of hidden structures, evaluation of dynamic performance and behaviors, and development of adaptive solutions that respond effectively to evolving conditions.

This book is motivated by the need to bridge theoretical insights in network controllability with simulation-based studies and practical applications across diverse domains, by incorporating advances in graph learning approaches and using tools from data-driven methodologies. A unified framework for controllability robustness analysis is introduced, integrating analytical, simulation-based, and data-driven approaches for measurement and analysis, together with optimization using computational intelligence techniques. This self-contained framework provides an efficient in-depth exploration of controllability, robustness, resilience, and adaptability in complex networks.

By integrating graph theory, optimization techniques, and machine learning algorithms, this book equips readers with useful tools to address real-world challenges in network functionality robustness. The content begins with foundational concepts and progresses through theoretical analysis, practical strategies for robustness evaluation and enhancement, applications of machine learning, and future research directions. This book is intended for researchers, practitioners and students in network science, control theory, optimization, AI-assisted systems engineering, and related fields. The goal is to provide readers with a comprehensive understanding of the structural and functional properties of complex networks and to enable the design, analysis and control of these systems in an increasingly interconnected and dynamic environment, with emphasis on network controllability and its robustness.

1.4.2 *Outline of the Book*

This book is structured into seven chapters, each focusing on a specific aspect of network controllability and its robustness. The remainder of the monograph is organized as follows:

- Chapter 2 introduces the basic concept and knowledge of network controllability, covering definitions, computational methods, and evaluation metrics. The chapter analyzes controllability across synthetic and real-world networks, explores key topological features, and distinguishes the controllability robustness from the connectivity robustness.
- Chapter 3 analyzes the controllability robustness in complex networks, introducing key metrics, attack strategies, hierarchical attack methods, simulation criteria, and analytical models. Comparative studies on synthetic and real-world networks will be performed to reveal how topology and heterogeneity influence resilience of complex networks.
- Chapter 4 explores techniques for enhancing the controllability robustness, introducing robustness-oriented models, metaheuristic-based optimization, and an empirical necessary condition verified through extensive experiments. The chapter demonstrates that edge rectification transforms networks toward highly homogeneous structures, providing a practical pathway for improved network robustness in design and applications.
- Chapter 5 examines data-driven approaches for evaluating the controllability robustness, focusing on input representation, model architecture, and output interpretation. Experiments are performed to show that structure-preserving representations and advanced graph neural network (GNN) models deliver the most accurate and reliable performances compared to other approaches.
- Chapter 6 introduces a framework for assessing and visualizing the controllability robustness enhancement potential, leveraging data-driven methods to deliver accurate predictions and the interpretability at significantly lower computational cost, compared to state-of-the-art approaches.
- Chapter 7 reviews recent advancements, identifies key challenges, and outlines future directions in network controllability robustness studies, emphasizing analytical foundations, simulation frameworks, and enhancement strategies.

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Appendix A

Lists of Symbols, Notation, Acronyms, Abbreviations

List of Symbols and Notation

- $A_{i,j}$: Entry of adjacency matrix
- $\bar{\xi}$: Mean prediction error
- \mathbf{A} : System matrix
- $\mathbf{A}^{(k)}$: High-order adjacency matrix
- \mathbf{B} : Input/Incidence matrix
- \mathbf{C} : Controllability matrix
- \mathbf{D} : Degree matrix
- \mathbf{H} : Modularity matrix
- \mathbf{L} : Laplacian matrix
- \mathbf{L}^* : Laplacian matrix with one row and column removed
- $\mathbf{W}^{(\ell)}$: Weight matrix at layer ℓ
- γ : Power-law exponent
- λ_{\max} : Largest eigenvalue
- $\langle k \rangle$: Average degree
- $\langle m \rangle$: Average number of edges
- $\langle n \rangle$: Average number of nodes
- n_D : Number of driver nodes
- \mathbb{R} : Real number field
- \mathcal{E}^* : Maximum matching set
- $|\mathcal{E}^*|$: Number of edges in maximum matching
- \mathcal{F} : Feature/Importance metric
- \mathcal{G}^* : Optimal network instance
- $\hat{\mathcal{G}}^*$: Approximate optimal network
- \mathcal{L} : Loss function
- σ : Proportion of nodes/edges been attacked

- $\sigma(\cdot)$: Activation function
- $\xi(i)$: Error at step i
- d_{st} : Shortest-path distance between nodes s, t
- $h_v^{(\ell)}$: Node representation at layer ℓ
- $h_{\mathcal{N}_v}^{(\ell+1)}$: Aggregated neighborhood representation
- H_G : Graph-level representation
- k_i : Degree of node i
- k_i^{in} : In-degree of node i
- k_i^{out} : Out-degree of node i
- L : Total number of layers
- m : Number of edges
- m_c : Number of critical edges
- n : Number of nodes
- c_D : Ratio and minimum number of driver nodes
- n_D : Minimum number of driver nodes
- $n_{\mathcal{N}}$: Number of nearest neighbors
- p_c : Connection or rewiring probability
- q : Snapback probability
- q_{QS} : Snapback probability in QS model
- r_{QS} : Layer index in QS model

List of Acronyms and Abbreviations

- ACO: Algebraic Connectivity
- APL: Average Path Length
- AST: Assortativity
- BDC: Betweenness-Clustering-Degree Embedding
- CNN: Convolutional Neural Network
- DeepWalk: Random Walk-Based Embedding Method
- DIA: Network Diameter
- DT: Decision Tree
- EFF: Efficiency
- EFR: Effective Resistance
- GAP: Spectral Gap
- GAT: Graph Attention Network
- GCN: Graph Convolutional Network
- GIN: Graph Isomorphism Network
- GNN: Graph Neural Network
- KNN: K -Nearest Neighbors

- LDP: Local Degree Profile
- LFR: Learning Feature Representation
- LINE: Large-Scale Information Network Embedding
- LR: Linear Regression
- LGC: Largest Circle
- MLP: Multi-Layer Perceptron
- MOD: Modularity
- mCNN-RP: Multi-Expert CNN Robustness Predictor
- NCO: Natural Connectivity
- NOC: Number of Circles
- NST: Number of Spanning Trees
- PATCHY-SAN: CNN-Based Graph Representation Method
- PCA: Principal Component Analysis
- PGR: PageRank
- RAD: Spectral Radius
- RF: Random Forest
- RPE: Robustness Potential Explorer
- RPE-F: Feature Extraction Module in RPE
- RPE-P: Prediction Module in RPE
- RPE-V: Visualization Module in RPE
- SDNE: Structural Deep Network Embedding
- SOM: Self-Organizing Map
- SPP: Spatial Pyramid Pooling
- struc2vec: Structural Role-Based Embedding Method
- t-SNE: t-Distributed Stochastic Neighbor Embedding
- TRA: Transitivity
- UMAP: Uniform Manifold Approximation and Projection

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Appendix B

Lists of Figures and Tables

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