Supplementary Information for the Paper Entitled "Predicting Network Controllability Robustness: A Convolutional Neural Network Approach"

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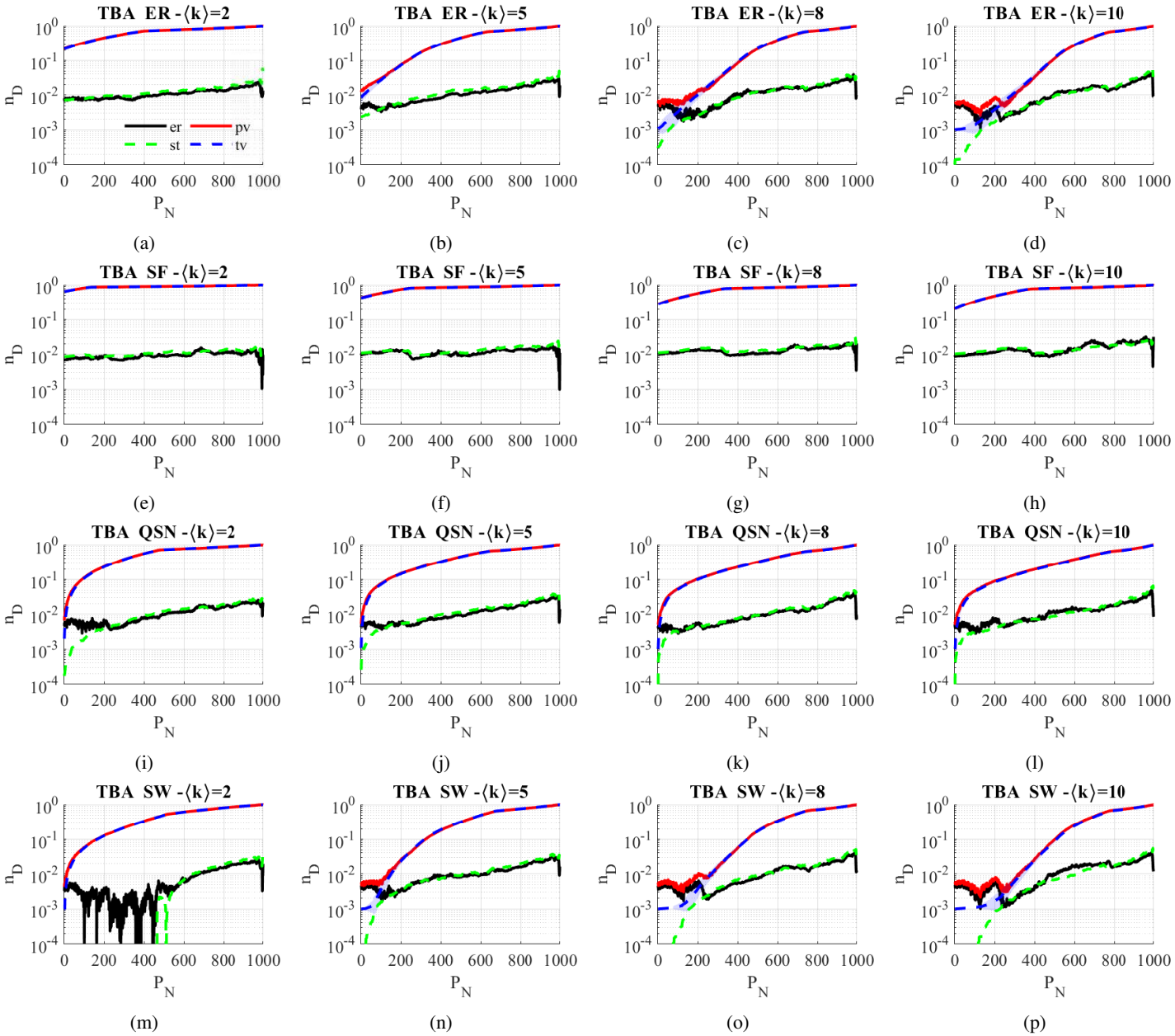

Figure S1: [Color online] Results of CNN controllability curve prediction under targeted betweenness-based attacks. P_N represents the number of nodes having been removed from the network; n_D is calculated by $n_D \equiv N_D/N$, where N is the current network size; $N_D = \max\{1, N - |E^*|\}$, with $|E^*|$ is the cardinal number of elements in the maximum matching E^*.

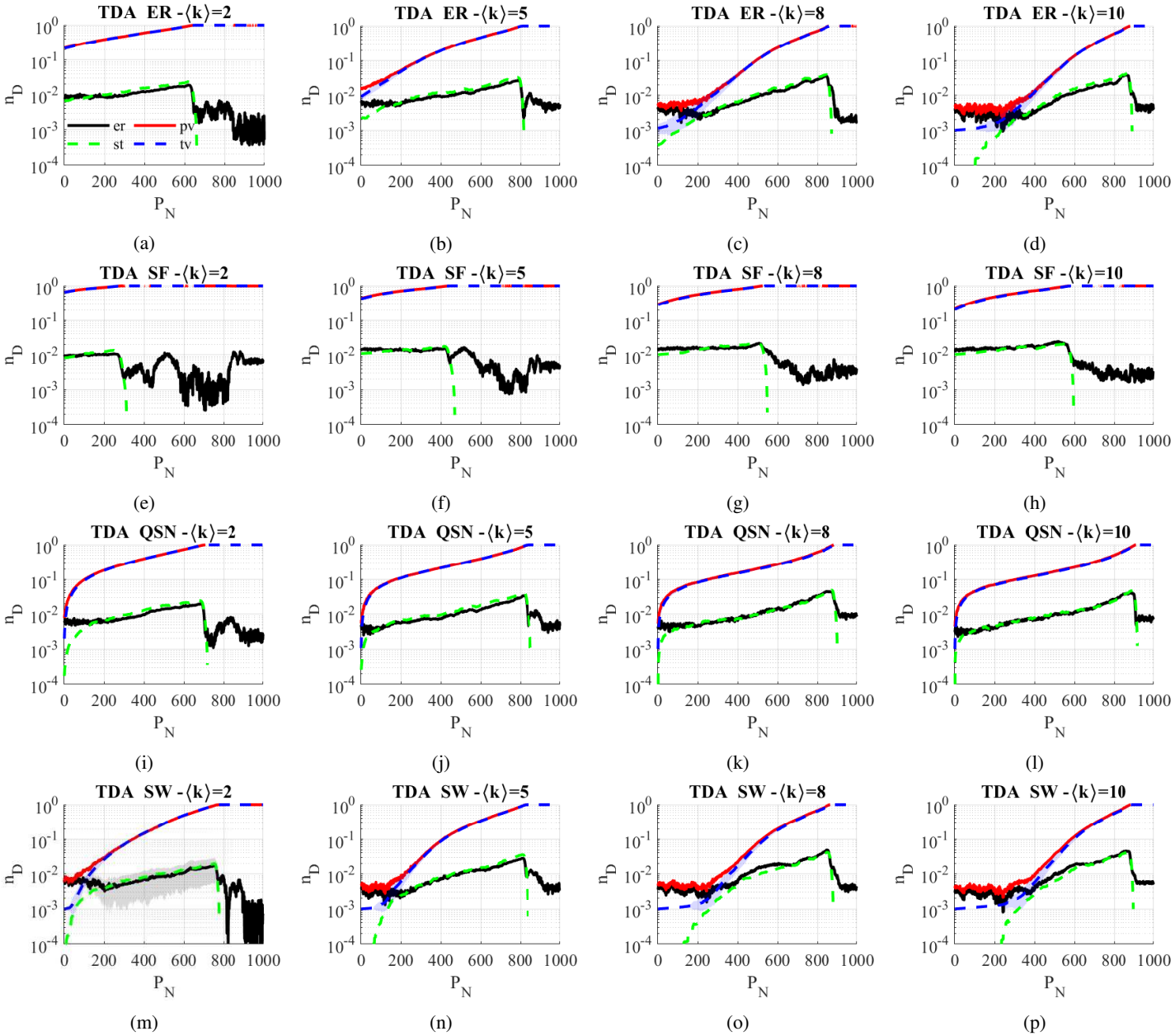

Figure S2: [Color online] Results of CNN controllability curve prediction under targeted degree-based attacks. P_N represents the number of nodes having been removed from the network; n_D is calculated by $n_D \equiv N_D/N$, where N is the current network size; $N_D = \max\{1, N - |E^*|\}$, with $|E^*|$ is the cardinal number of elements in the maximum matching E^*.

Table S1: The mean error of prediction vs. the mean standard deviation of the testing data for the unweighted networks under TBA and TDA.

			$\langle k\rangle = 2$	$\langle k\rangle = 5$	$\langle k\rangle = 8$	$\langle k\rangle = 10$
TBA	ER	$\bar{er}$	0.011	0.012	0.012	0.012
		$\bar{\sigma}$	0.014	0.014	0.013	0.012
	SF	$\bar{er}$	0.009	0.012	0.013	0.015
		$\bar{\sigma}$	0.011	0.013	0.015	0.015
	QSN	$\bar{er}$	0.011	0.012	0.011	0.012
		$\bar{\sigma}$	0.012	0.014	0.013	0.013
	SW	$\bar{er}$	0.008	0.011	0.011	0.012
		$\bar{\sigma}$	0.008	0.012	0.012	0.011
TDA	ER	$\bar{er}$	0.008	0.010	0.010	0.009
		$\bar{\sigma}$	0.009	0.011	0.010	0.010
	SF	$\bar{er}$	0.006	0.009	0.011	0.011
		$\bar{\sigma}$	0.003	0.006	0.008	0.009
	QSN	$\bar{er}$	0.008	0.011	0.013	0.012
		$\bar{\sigma}$	0.008	0.012	0.012	0.011
	SW	$\bar{er}$	0.007	0.009	0.013	0.012
		$\bar{\sigma}$	0.006	0.009	0.009	0.009

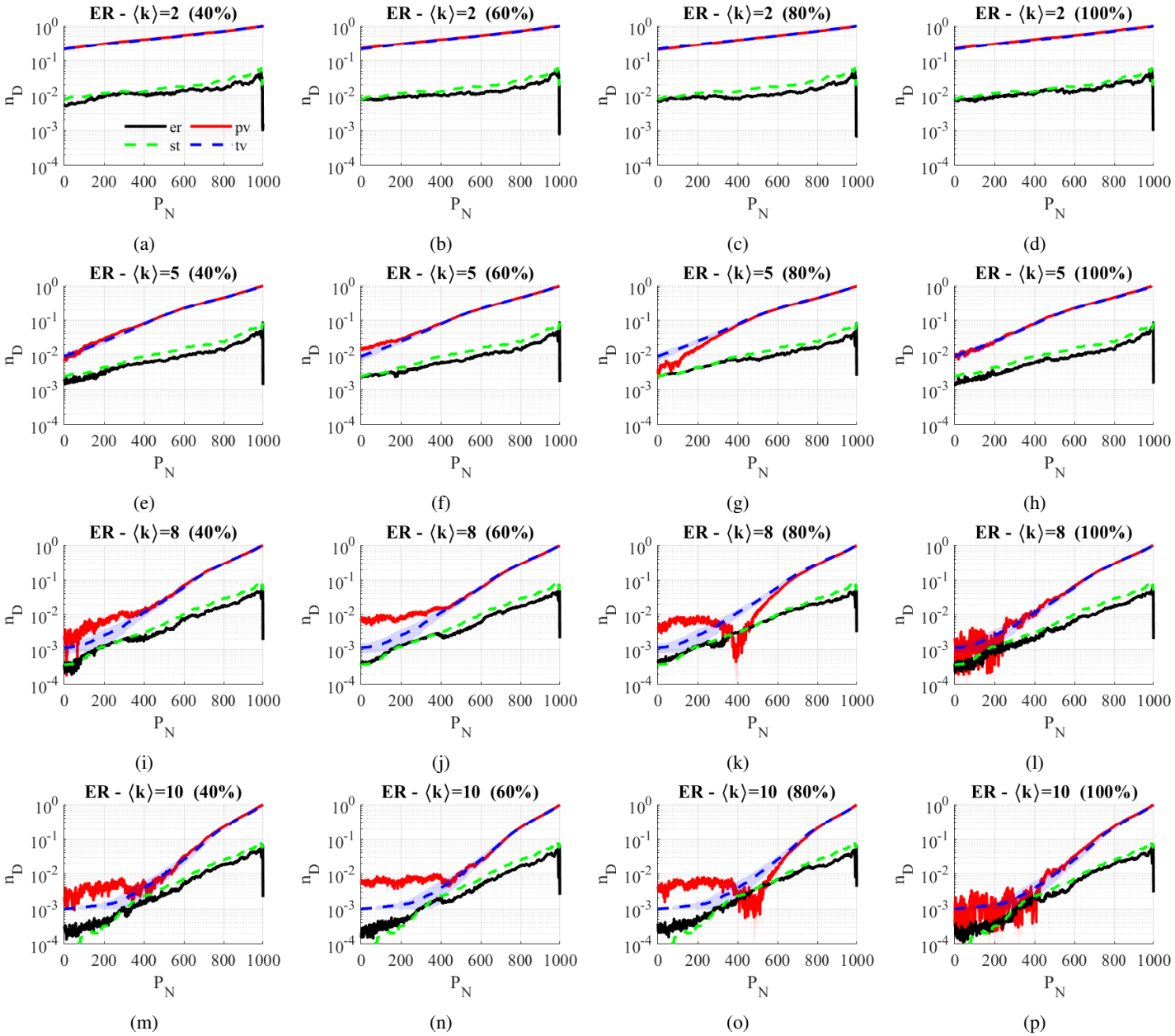

Figure S3: [Color online] Results of CNN controllability curve prediction under random attacks on ER networks. The size of training data is set to 40%, 60%, 80%, and 100% of the data size used in the paper, where 100% represents a training set of 800 instances.

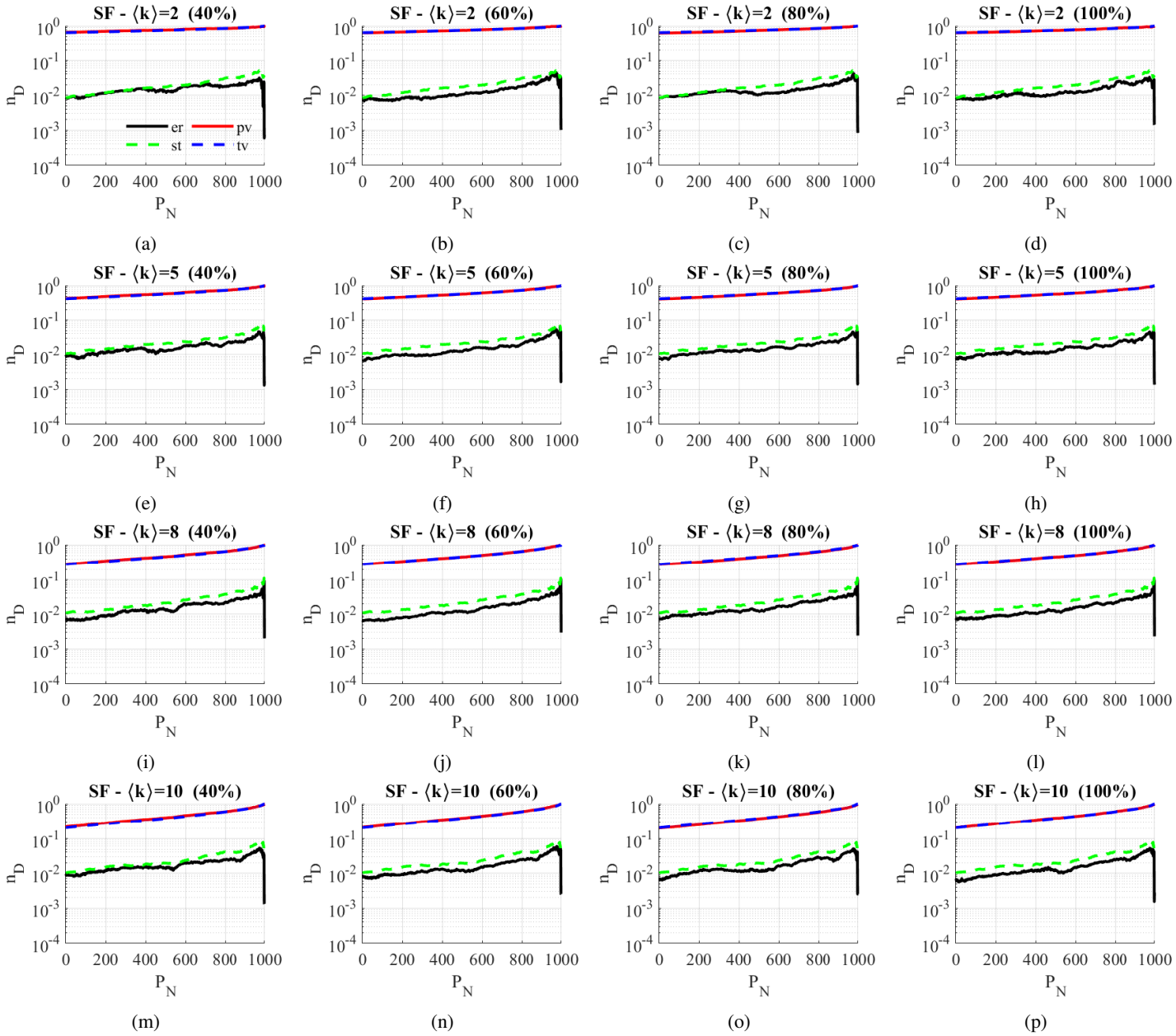

Figure S4: [Color online] Results of CNN controllability curve prediction under random attacks on SF networks. The size of training data is set to 40%, 60%, 80%, and 100% of the data size used in the paper, where 100% represents a training set of 800 instances.

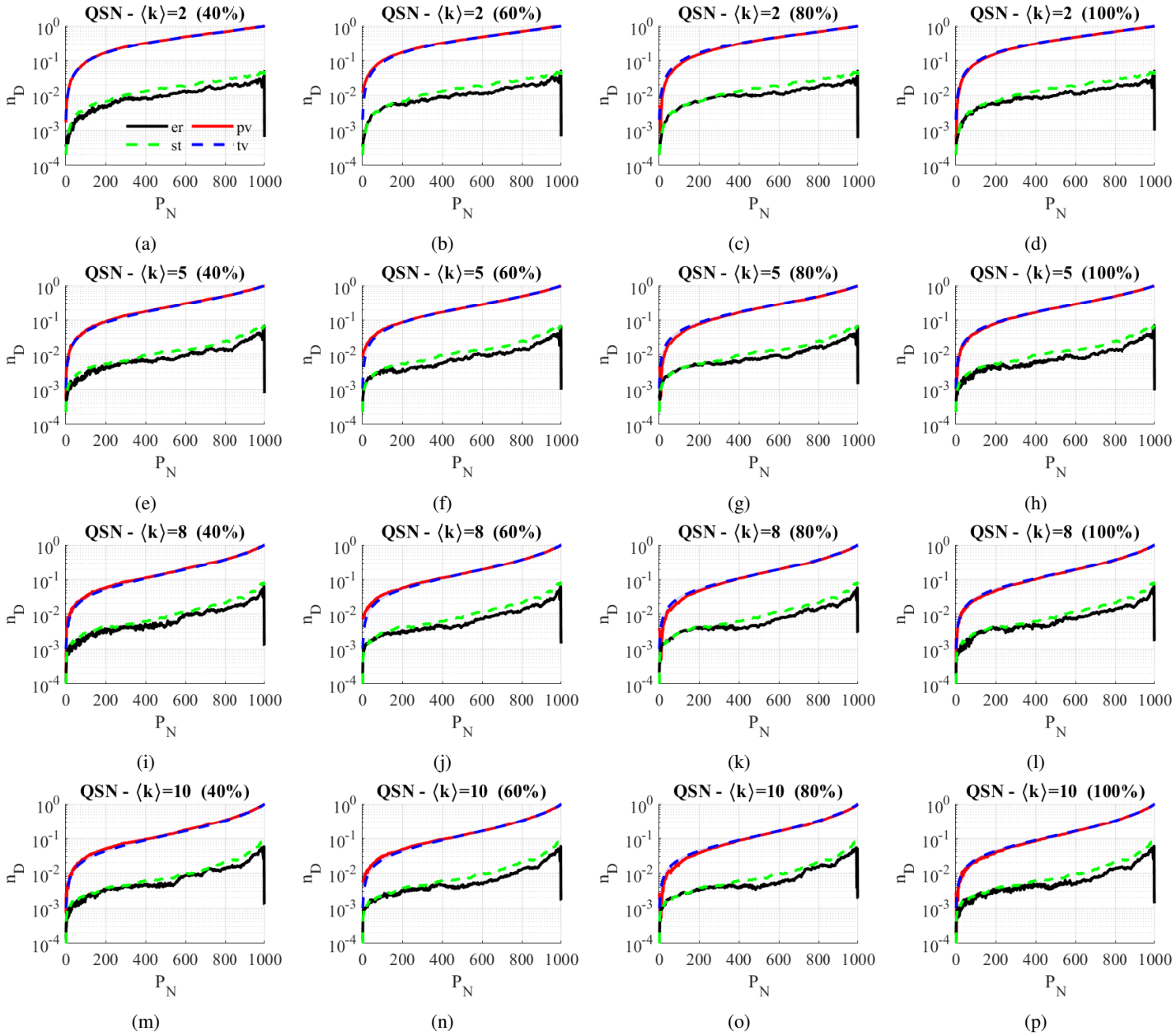

Figure S5: [Color online] Results of CNN controllability curve prediction under random attacks on QSN networks. The size of training data is set to 40%, 60%, 80%, and 100% of the data size used in the paper, where 100% represents a training set of 800 instances.

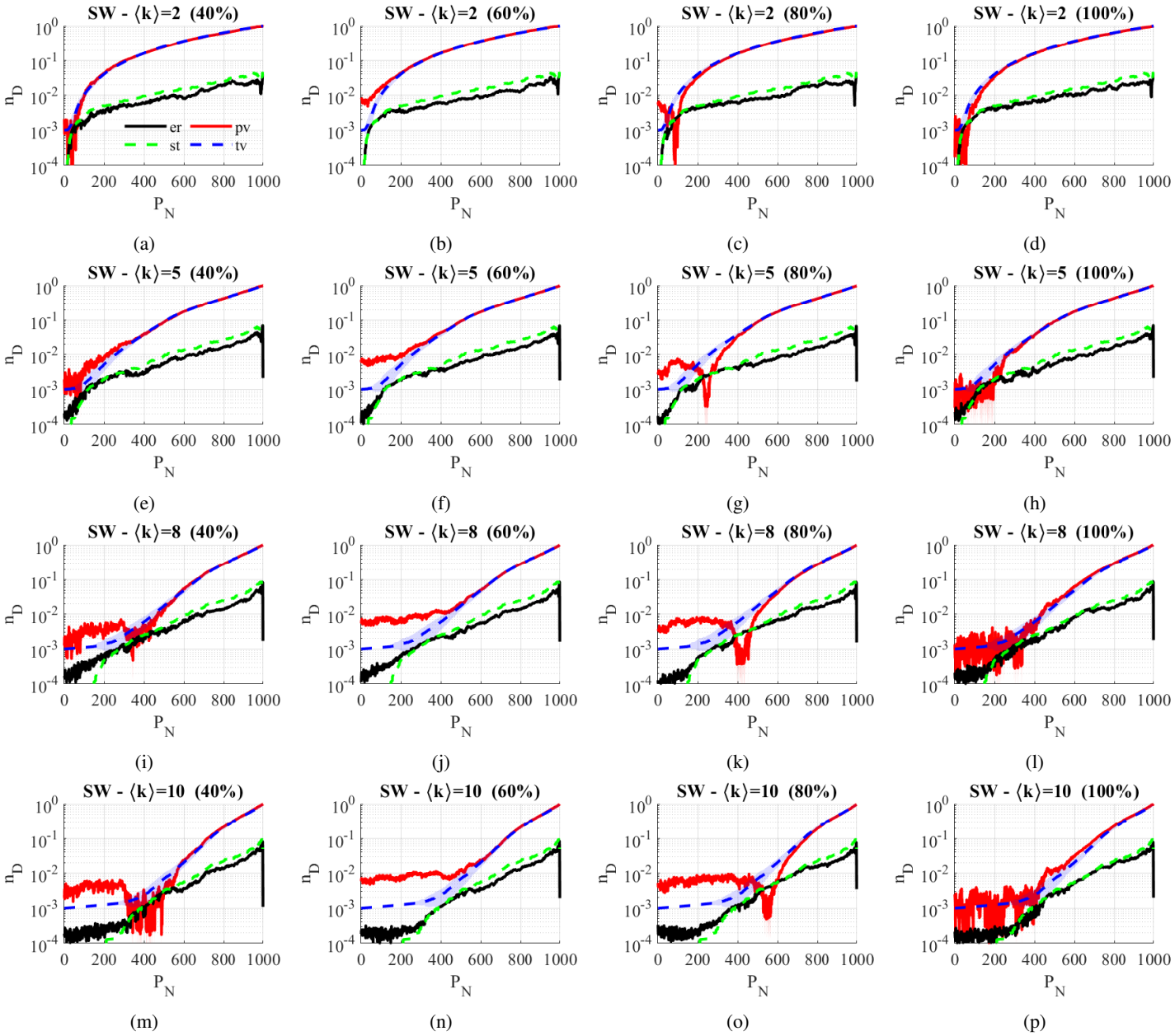

Figure S6: [Color online] Results of CNN controllability curve prediction under random attacks on SW networks. The size of training data is set to 40%, 60%, 80%, and 100% of the data size used in the paper, where 100% represents a training set of 800 instances.

Table S2: The mean error of prediction vs. the mean standard deviation of the testing data for the unweighted networks ER, SF, QSN, and SW. The size of training data is set to 40%, 60%, 80%, and 100% respectively, where 100% represents a training set of 800 instances.

data size			40%	60%	80%	100%
ER (see Fig.S3)	$\langle k\rangle = 2$	$\bar{er}$	0.0135	0.0137	0.0125	0.0147
		$\bar{\sigma}$	0.0191	0.0191	0.0191	0.0191
	$\langle k\rangle = 5$	$\bar{er}$	0.0108	0.0108	0.0113	0.0112
		$\bar{\sigma}$	0.0169	0.0169	0.0169	0.0169
	$\langle k\rangle = 8$	$\bar{er}$	0.0095	0.0093	0.0105	0.0093
		$\bar{\sigma}$	0.0146	0.0146	0.0146	0.0146
	$\langle k\rangle = 10$	$\bar{er}$	0.0091	0.0081	0.0086	0.0091
		$\bar{\sigma}$	0.0128	0.0128	0.0128	0.0128
SF (see Fig.S4)	$\langle k\rangle = 2$	$\bar{er}$	0.0156	0.0135	0.0144	0.0136
		$\bar{\sigma}$	0.0206	0.0206	0.0206	0.0206
	$\langle k\rangle = 5$	$\bar{er}$	0.0168	0.0160	0.0163	0.0159
		$\bar{\sigma}$	0.0243	0.0243	0.0243	0.0243
	$\langle k\rangle = 8$	$\bar{er}$	0.0169	0.0161	0.0174	0.0161
		$\bar{\sigma}$	0.0265	0.0265	0.0265	0.0265
	$\langle k\rangle = 10$	$\bar{er}$	0.0184	0.0171	0.0174	0.0163
		$\bar{\sigma}$	0.0264	0.0264	0.0264	0.0264
QSN (see Fig.S5)	$\langle k\rangle = 2$	$\bar{er}$	0.0113	0.0119	0.0119	0.0114
		$\bar{\sigma}$	0.0176	0.0176	0.0176	0.0176
	$\langle k\rangle = 5$	$\bar{er}$	0.0108	0.0102	0.0108	0.0101
		$\bar{\sigma}$	0.0160	0.0160	0.0160	0.0160
	$\langle k\rangle = 8$	$\bar{er}$	0.0093	0.0088	0.0091	0.0092
		$\bar{\sigma}$	0.0140	0.0140	0.0140	0.0140
	$\langle k\rangle = 10$	$\bar{er}$	0.0093	0.0083	0.0090	0.0084
		$\bar{\sigma}$	0.0132	0.0132	0.0132	0.0132
SW (see Fig.S6)	$\langle k\rangle = 2$	$\bar{er}$	0.0101	0.0095	0.0103	0.0101
		$\bar{\sigma}$	0.0152	0.0152	0.0152	0.0152
	$\langle k\rangle = 5$	$\bar{er}$	0.0096	0.0094	0.0094	0.0095
		$\bar{\sigma}$	0.0149	0.0149	0.0149	0.0149
	$\langle k\rangle = 8$	$\bar{er}$	0.0088	0.0085	0.0089	0.0098
		$\bar{\sigma}$	0.0137	0.0137	0.0137	0.0137
	$\langle k\rangle = 10$	$\bar{er}$	0.0080	0.0075	0.0081	0.0091
		$\bar{\sigma}$	0.0118	0.0118	0.0118	0.0118