Supplementary Information for the Paper "Enhancing Controllability Robustness of q-snapback Networks Through Re-directing Edges"

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Contents

1 Expected Out-degrees and In-degrees

In Fig. S1, the expected out-degree and in-degree for each node calculated according to Eqs. (1) and (2) are plotted together with the real degrees of each node averaged from 1000 independent runs. The network size is $N = 1000$. It can be seen from Fig. S1 that the expected out-degree and in-degree of each node can be precisely calculated by Eqs. (1) and (2), respectively.

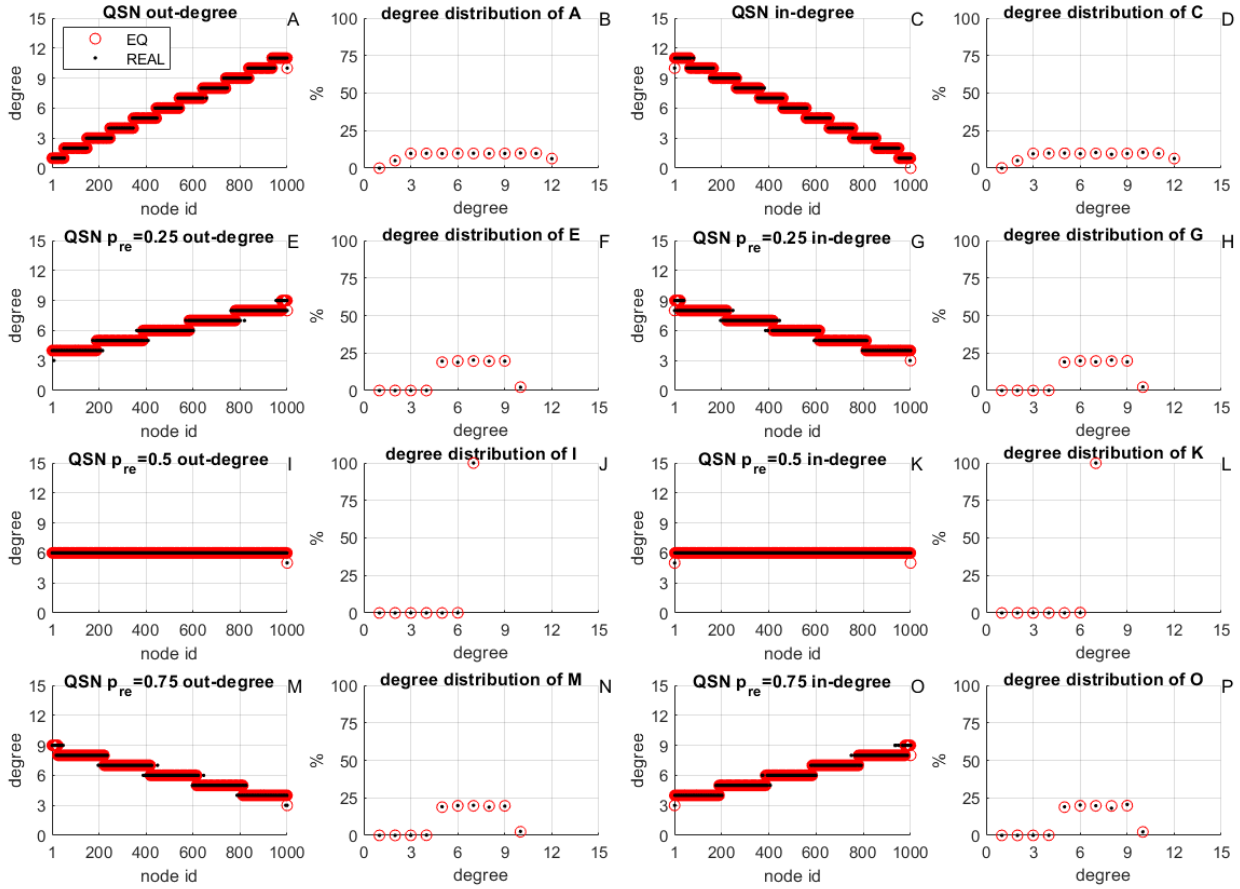


Figure S1 (A, C, E, G, I, K, M, and O) The expected out-degree and in-degree for node id (1,2, ..., 1000) is plotted in red circle, and the real degree (averaged from 1000 independent runs) is plotted in black dot. (B, D, F, H, J, L, N, and P) The according degree distribution of its left figure.

2 QSN vs. QSN Variants Comparison Tables

The comparisons of the original q-snapback network (QSN) and the QSN variants (with re-directed edges) are given in Tables. S1 to S16. The network size is set to $N = 500$, $N = 1000$, and $N = 2000$, respectively. For $N = 500$, the average degree of networks is set to $\langle k \rangle = 5.38$ and $\langle k \rangle = 10$, respectively. For $N = 1000$, the average degree is set to $\langle k \rangle = \{6.069, 10, 20\}$, respectively. For $N = 2000$, the average degree is set to $\langle k \rangle = \{6.759, 10, 20\}$, respectively. Both exact controllability (EC) and structural controllability (SC) are compared. Note that when $N = 500$ and $\langle k \rangle = 20$, then the ratio between the number of edges M and possible maximum number of edges M_{max} is $M/M_{max} = 0.08 > 0.05$, meaning it is not a sparse

network and the calculation of exact controllability cannot be applied to it, and thus excluded from the comparison.

In each cell of the table, the real number represents the average rank, and the integer inside the parentheses mean the number of winning times. Bold real numbers (with gray-shaded) represent the minimum average rank, and bold numbers inside parentheses (with gray-shaded) mean the maximum average number of winning times.

2.1 Network Size N=500

Table S1. $[N = 500, \langle k \rangle = 5.38]$ Comparison of the original QSN and the QSN with re-directed edges in terms of exact controllability (EC).

EC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
QSN	10.49 (2)	7.33 (66)	8.06 (74)	10.11 (4)	8.75 (88)	7.39 (425)	8.69 (110)
QSN $p_{re} = 0.1$	8.37 (2)	7.34 (21)	5.48 (180)	8.12 (9)	8.34 (90)	8.51 (93)	7.69 (66)
QSN $p_{re} = 0.2$	6.04 (9)	6.82 (29)	6.73 (75)	6.34 (65)	7.37 (19)	6.36 (76)	6.61 (46)
QSN $p_{re} = 0.3$	3.96 (13)	4.41 (172)	4.14 (142)	4.17 (168)	3.31 (406)	5.99 (128)	4.33 (172)
QSN $p_{re} = 0.4$	1.95 (193)	3.81 (138)	3.61 (244)	2.97 (278)	4.34 (233)	2.40 (743)	**3.18** (305)
QSN $p_{re} = 0.5$	2.03 (238)	4.95 (27)	6.42 (167)	1.39 (2148)	3.01 (1384)	2.66 (1056)	3.41 **(837)**
QSN $p_{re} = 0.6$	3.70 (76)	4.99 (101)	4.33 (140)	2.77 (396)	4.24 (609)	2.91 (974)	3.82 (383)
QSN $p_{re} = 0.7$	5.13 (6)	6.06 (30)	6.38 (90)	4.56 (8)	4.15 (369)	4.64 (89)	5.15 (99)
QSN $p_{re} = 0.8$	5.85 (12)	7.22 (56)	7.39 (89)	6.65 (22)	6.31 (90)	6.45 (30)	6.65 (50)
QSN $p_{re} = 0.9$	8.72 (2)	4.97 (139)	5.37 (144)	8.27 (30)	7.22 (92)	9.28 (54)	7.30 (77)
QSN $p_{re} = 1.0$	9.75 (1)	8.10 (29)	8.09 (92)	10.65 (13)	8.98 (19)	9.40 (44)	9.16 (33)

Table S2. $[N = 500, \langle k \rangle = 5.38]$ Comparison of the original QSN and the QSN with re-directed edges in terms of structural controllability (SC).

SC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
QSN	10.50 (2)	7.35 (66)	8.06 (74)	10.12 (4)	8.81 (88)	7.40 (425)	8.71 (110)
QSN $p_{re} = 0.1$	8.37 (2)	7.34 (21)	5.49 (179)	8.11 (9)	8.25 (90)	8.52 (93)	7.68 (66)
QSN $p_{re} = 0.2$	6.05 (9)	6.83 (29)	6.73 (75)	6.33 (65)	7.41 (21)	6.35 (76)	6.62 (46)
QSN $p_{re} = 0.3$	3.95 (13)	4.42 (172)	4.14 (142)	4.15 (168)	3.32 (382)	6.00 (128)	4.33 (168)
QSN $p_{re} = 0.4$	1.93 (192)	3.83 (137)	3.62 (236)	2.99 (278)	4.27 (253)	2.41 (714)	**3.17** (302)
QSN $p_{re} = 0.5$	1.98 (247)	4.94 (35)	6.43 (167)	1.39 (2153)	3.02 (1363)	2.68 (1037)	3.41 **(834)**
QSN $p_{re} = 0.6$	3.78 (67)	5.00 (100)	4.33 (142)	2.76 (389)	4.27 (611)	2.90 (989)	3.84 (383)
QSN $p_{re} = 0.7$	5.15 (6)	6.06 (30)	6.39 (89)	4.58 (8)	4.16 (359)	4.63 (89)	5.16 (97)
QSN $p_{re} = 0.8$	5.82 (12)	7.21 (56)	7.31 (89)	6.66 (22)	6.31 (90)	6.44 (30)	6.62 (50)
QSN $p_{re} = 0.9$	8.72 (2)	4.98 (139)	5.42 (144)	8.27 (30)	7.22 (92)	9.27 (54)	7.32 (77)
QSN $p_{re} = 1.0$	9.74 (1)	8.04 (37)	8.09 (92)	10.64 (13)	8.96 (19)	9.41 (44)	9.15 (34)

Table S3. $[N = 500, \langle k \rangle = 10]$ Comparison of the original QSN and the QSN with re-directed edges in terms of exact controllability (EC).

EC	**R_N**	**TB_N**	**TD_N**	**R_E**	**TB_E**	**TD_E**	**Average**
QSN	10.50 (3)	9.33 (11)	8.33 (60)	10.65 (21)	8.58 (49)	10.59 (17)	9.66 (27)
QSN $p_{re} = 0.1$	7.81 (50)	6.97 (59)	6.34 (150)	8.53 (200)	8.59 (392)	8.40 (393)	7.77 (207)
QSN $p_{re} = 0.2$	6.26 (40)	5.66 (70)	3.94 (289)	6.71 (449)	6.28 (195)	5.11 (848)	5.66 (315)
QSN $p_{re} = 0.3$	4.57 (81)	4.82 (90)	7.00 (132)	4.03 (1438)	3.49 (756)	5.00 (829)	4.82 (554)
QSN $p_{re} = 0.4$	3.23 (137)	4.41 (219)	4.91 (273)	2.86 (1990)	5.73 (181)	2.88 (1487)	4.00 (715)
QSN $p_{re} = 0.5$	1.83 (437)	4.29 (236)	5.13 (225)	2.04 (3598)	2.53 (2929)	2.59 (2840)	**3.07 (1711)**
QSN $p_{re} = 0.6$	3.10 (184)	4.53 (141)	4.07 (273)	2.41 (2415)	2.94 (697)	2.38 (2647)	3.24 (1060)
QSN $p_{re} = 0.7$	4.21 (114)	4.06 (149)	5.61 (167)	4.34 (1157)	5.20 (714)	4.65 (996)	4.68 (550)
QSN $p_{re} = 0.8$	6.23 (39)	6.59 (27)	5.09 (168)	5.95 (507)	4.99 (439)	6.52 (579)	5.90 (293)
QSN $p_{re} = 0.9$	8.31 (14)	6.74 (64)	6.79 (61)	8.32 (36)	8.22 (65)	8.24 (425)	7.77 (111)
QSN $p_{re} = 1.0$	9.96 (1)	8.61 (23)	8.79 (42)	10.16 (5)	9.44 (18)	9.63 (8)	9.43 (16)

Table S4. $[N = 500, \langle k \rangle = 10]$ Comparison of the original QSN and the QSN with re-directed edges in terms of structural controllability (SC).

SC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
QSN	10.52 (3)	9.31 (13)	8.36 (57)	10.63 (21)	8.54 (49)	10.59 (17)	9.66 (27)
QSN $p_{re} = 0.1$	7.81 (50)	6.99 (59)	6.35 (148)	8.53 (200)	8.59 (399)	8.40 (394)	7.78 (208)
QSN $p_{re} = 0.2$	6.27 (40)	5.71 (70)	3.95 (285)	6.71 (449)	6.30 (195)	5.10 (848)	5.67 (315)
QSN $p_{re} = 0.3$	4.57 (81)	4.86 (90)	7.01 (132)	4.02 (1438)	3.54 (624)	5.00 (829)	4.84 (532)
QSN $p_{re} = 0.4$	3.21 (137)	4.49 (205)	4.91 (274)	2.86 (2000)	5.74 (225)	2.89 (1478)	4.02 (720)
QSN $p_{re} = 0.5$	1.82 (440)	4.30 (248)	5.16 (222)	2.07 (3463)	2.55 (2907)	2.58 (2832)	**3.08 (1685)**
QSN $p_{re} = 0.6$	3.12 (183)	4.56 (146)	4.04 (277)	2.38 (2547)	2.98 (634)	2.38 (2656)	3.24 (1074)
QSN $p_{re} = 0.7$	4.21 (113)	4.10 (149)	5.56 (170)	4.35 (1157)	5.14 (728)	4.66 (996)	4.67 (552)
QSN $p_{re} = 0.8$	6.23 (38)	6.64 (27)	5.07 (164)	5.96 (507)	4.94 (593)	6.53 (579)	5.90 (318)
QSN $p_{re} = 0.9$	8.31 (14)	6.43 (102)	6.80 (61)	8.31 (36)	8.22 (65)	8.22 (451)	7.71 (122)
QSN $p_{re} = 1.0$	9.94 (1)	8.61 (23)	8.79 (42)	10.18 (5)	9.47 (18)	9.63 (8)	9.44 (16)

2.2 Network Size N=1000

Table S5. $[N = 1000, \langle k \rangle = 6.069]$ Comparison of the original QSN and the QSN with re-directed edges in terms of exact controllability (EC).

EC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
QSN	10.04 (30)	7.30 (125)	7.79 (132)	10.57 (10)	8.55 (282)	9.98 (24)	9.04 (101)
QSN $p_{re} = 0.1$	8.40 (4)	6.89 (92)	6.05 (157)	8.36 (41)	7.31 (390)	8.61 (295)	7.60 (163)
QSN $p_{re} = 0.2$	5.88 (9)	7.21 (32)	5.26 (156)	6.45 (111)	6.50 (225)	6.93 (150)	6.37 (114)
QSN $p_{re} = 0.3$	4.16 (21)	5.10 (77)	5.99 (218)	4.58 (111)	4.43 (633)	4.92 (474)	4.86 (256)
QSN $p_{re} = 0.4$	2.79 (75)	4.24 (157)	5.33 (309)	2.23 (1410)	3.16 (667)	2.33 (2034)	3.34 (775)
QSN $p_{re} = 0.5$	1.40 (883)	3.90 (260)	5.15 (370)	1.69 (4218)	3.59 (2912)	2.84 (2157)	**3.09 (1800)**
QSN $p_{re} = 0.6$	3.07 (14)	4.24 (459)	6.00 (175)	2.44 (854)	3.83 (482)	2.61 (2544)	3.70 (755)
QSN $p_{re} = 0.7$	4.83 (16)	5.11 (117)	4.14 (378)	4.33 (288)	3.65 (1320)	4.03 (281)	4.35 (400)
QSN $p_{re} = 0.8$	6.62 (13)	5.42 (88)	4.99 (236)	6.62 (46)	7.13 (123)	5.81 (351)	6.10 (143)
QSN $p_{re} = 0.9$	8.57 (3)	7.83 (33)	8.15 (135)	8.42 (30)	8.92 (233)	9.22 (106)	8.52 (90)
QSN $p_{re} = 1.0$	10.25 (8)	8.76 (37)	7.15 (348)	10.31 (11)	8.94 (94)	8.73 (85)	9.02 (97)

Table S6. $[N = 1000, \langle k \rangle = 6.069]$ Comparison of the original QSN and the QSN with re-directed edges in terms of structural controllability (SC).

SC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
QSN	10.04 (31)	7.30 (125)	7.79 (132)	10.58 (14)	8.52 (283)	9.98 (25)	9.03 (102)
QSN $p_{re} = 0.1$	8.39 (4)	6.89 (92)	6.05 (157)	8.36 (41)	7.28 (454)	8.61 (295)	7.60 (174)
QSN $p_{re} = 0.2$	5.88 (9)	7.20 (32)	5.26 (156)	6.45 (111)	6.49 (225)	6.92 (150)	6.37 (114)
QSN $p_{re} = 0.3$	4.16 (21)	5.11 (77)	5.99 (218)	4.58 (111)	4.43 (705)	4.92 (474)	4.86 (268)
QSN $p_{re} = 0.4$	2.79 (74)	4.24 (157)	5.31 (318)	2.22 (1410)	3.17 (690)	2.34 (1947)	3.34 (766)
QSN $p_{re} = 0.5$	1.40 (883)	3.90 (260)	5.15 (364)	1.69 (4219)	3.60 (2971)	2.83 (2227)	**3.09 (1821)**
QSN $p_{re} = 0.6$	3.09 (12)	4.24 (458)	6.00 (174)	2.44 (853)	3.86 (482)	2.61 (2575)	3.71 (759)
QSN $p_{re} = 0.7$	4.84 (16)	5.11 (117)	4.15 (376)	4.33 (288)	3.67 (1170)	4.03 (281)	4.36 (375)
QSN $p_{re} = 0.8$	6.61 (13)	5.42 (88)	4.99 (236)	6.62 (46)	7.13 (125)	5.81 (351)	6.10 (143)
QSN $p_{re} = 0.9$	8.57 (3)	7.83 (33)	8.15 (135)	8.42 (30)	8.89 (239)	9.22 (106)	8.51 (91)
QSN $p_{re} = 1.0$	10.25 (8)	8.76 (37)	7.16 (348)	10.29 (11)	8.96 (61)	8.73 (85)	9.03 (92)

Table S7. $[N = 1000, \langle k \rangle = 10]$ Comparison of the original QSN and the QSN with re-directed edges in terms of exact controllability (EC).

EC	**R_N**	**TB_N**	**TD_N**	**R_E**	**TB_E**	**TD_E**	**Average**
QSN	10.22 (2)	9.13 (13)	8.15 (266)	10.61 (22)	8.65 (83)	10.32 (34)	9.51 (70)
QSN $p_{re} = 0.1$	7.99 (11)	6.74 (50)	6.32 (216)	8.20 (242)	7.80 (535)	7.87 (405)	7.49 (243)
QSN $p_{re} = 0.2$	6.41 (47)	6.27 (92)	5.67 (265)	6.52 (487)	6.69 (328)	6.33 (1233)	6.31 (409)
QSN $p_{re} = 0.3$	4.07 (105)	3.93 (264)	4.68 (420)	4.66 (1404)	4.05 (602)	4.40 (590)	4.30 (564)
QSN $p_{re} = 0.4$	2.21 (474)	5.29 (210)	4.84 (566)	2.57 (3321)	3.61 (2827)	2.66 (1230)	3.53 (1438)
QSN $p_{re} = 0.5$	2.50 (373)	3.22 (584)	4.92 (408)	2.02 (7349)	4.59 (3047)	1.53 (9364)	**3.13 (3521)**
QSN $p_{re} = 0.6$	2.46 (518)	4.04 (365)	5.19 (304)	2.54 (3349)	4.10 (211)	2.99 (963)	3.55 (952)
QSN $p_{re} = 0.7$	4.83 (71)	5.23 (52)	5.08 (339)	3.98 (2152)	4.80 (1160)	5.26 (470)	4.86 (707)
QSN $p_{re} = 0.8$	6.18 (75)	6.58 (116)	4.98 (314)	5.96 (1527)	6.29 (1618)	5.90 (535)	5.98 (698)
QSN $p_{re} = 0.9$	8.61 (15)	7.57 (52)	7.60 (132)	8.61 (168)	6.89 (812)	8.37 (569)	7.94 (291)
QSN $p_{re} = 1.0$	10.53 (1)	7.99 (206)	8.58 (92)	10.35 (6)	8.54 (3)	10.37 (67)	9.39 (63)

Table S8. $[N = 1000, \langle k \rangle = 10]$ Comparison of the original QSN and the QSN with re-directed edges in terms of structural controllability (SC).

SC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
QSN	10.22 (2)	9.11 (14)	8.15 (266)	10.61 (22)	8.64 (83)	10.32 (37)	9.51 (71)
QSN $p_{re} = 0.1$	7.96 (11)	6.77 (50)	6.35 (211)	8.21 (242)	7.80 (539)	7.87 (405)	7.49 (243)
QSN $p_{re} = 0.2$	6.38 (48)	6.24 (92)	5.68 (263)	6.51 (487)	6.70 (328)	6.33 (1233)	6.31 (409)
QSN $p_{re} = 0.3$	4.08 (105)	3.94 (264)	4.67 (423)	4.65 (1404)	4.02 (716)	4.39 (590)	4.29 (584)
QSN $p_{re} = 0.4$	2.19 (481)	5.26 (210)	4.80 (566)	2.57 (3321)	3.59 (2797)	2.66 (1201)	3.51 (1429)
QSN $p_{re} = 0.5$	2.48 (382)	3.23 (584)	4.90 (409)	2.02 (7348)	4.61 (3109)	1.53 (9385)	**3.13 (3536)**
QSN $p_{re} = 0.6$	2.48 (502)	4.05 (366)	5.22 (304)	2.54 (3351)	4.11 (222)	2.99 (963)	3.57 (951)
QSN $p_{re} = 0.7$	4.83 (70)	5.24 (52)	5.08 (339)	3.98 (2152)	4.84 (1105)	5.27 (470)	4.87 (698)
QSN $p_{re} = 0.8$	6.21 (75)	6.59 (116)	4.99 (314)	5.97 (1527)	6.28 (1637)	5.90 (535)	5.99 (701)
QSN $p_{re} = 0.9$	8.64 (15)	7.58 (52)	7.58 (132)	8.60 (168)	6.90 (816)	8.37 (569)	7.94 (292)
QSN $p_{re} = 1.0$	10.53 (1)	7.99 (206)	8.58 (92)	10.34 (6)	8.52 (3)	10.37 (67)	9.39 (63)

Table S9. $[N = 1000, \langle k \rangle = 20]$ Comparison of the original QSN and the QSN with re-directed edges in terms of exact controllability (EC).

EC	**R_N**	**TB_N**	**TD_N**	**R_E**	**TB_E**	**TD_E**	**Average**
QSN	10.00 (4)	9.71 (22)	9.04 (109)	10.28 (34)	10.21 (18)	10.13 (35)	9.89 (37)
QSN $p_{re} = 0.1$	8.48 (95)	6.85 (199)	7.57 (154)	7.72 (2852)	7.68 (1183)	8.57 (1733)	7.81 (1036)
QSN $p_{re} = 0.2$	5.97 (318)	5.80 (226)	5.03 (483)	5.71 (7389)	4.89 (536)	6.44 (3280)	5.64 (2039)
QSN $p_{re} = 0.3$	4.02 (575)	4.14 (405)	4.88 (610)	4.33 (9175)	5.07 (1155)	4.39 (4746)	4.47 (2778)
QSN $p_{re} = 0.4$	3.30 (690)	4.08 (502)	5.08 (630)	2.95 (13729)	4.47 (360)	2.96 (6415)	3.81 (3721)
QSN $p_{re} = 0.5$	3.02 (813)	3.48 (801)	4.79 (684)	2.71 (18468)	2.49 (10989)	2.04 (15683)	**3.09 (7906)**
QSN $p_{re} = 0.6$	3.47 (667)	4.09 (342)	4.01 (784)	3.48 (10868)	3.25 (5058)	2.75 (9256)	3.51 (4496)
QSN $p_{re} = 0.7$	4.12 (585)	5.02 (398)	4.73 (581)	4.23 (10462)	4.00 (1213)	4.32 (4593)	4.40 (2972)
QSN $p_{re} = 0.8$	5.33 (482)	6.35 (87)	4.80 (532)	5.70 (7831)	7.61 (638)	5.85 (3752)	5.94 (2220)
QSN $p_{re} = 0.9$	7.58 (150)	7.54 (85)	6.22 (320)	8.20 (3274)	6.81 (522)	8.31 (1729)	7.44 (1013)
QSN $p_{re} = 1.0$	10.72 (2)	8.95 (120)	9.87 (47)	10.68 (13)	9.53 (13)	10.26 (9)	10.00 (34)

Table S10. $[N = 1000, \langle k \rangle = 20]$ Comparison of the original QSN and the QSN with re-directed edges in terms of structural controllability (SC).

SC	**R_N**	**TB_N**	**TD_N**	**R_E**	**TB_E**	**TD_E**	**Average**
QSN	10.01 (4)	9.70 (24)	9.06 (109)	10.30 (34)	10.21 (18)	10.12 (36)	9.90 (38)
QSN $p_{re} = 0.1$	8.49 (95)	6.87 (201)	7.58 (148)	7.72 (2852)	7.67 (1275)	8.57 (1733)	7.82 (1051)
QSN $p_{re} = 0.2$	5.96 (318)	5.80 (226)	5.04 (474)	5.71 (7389)	4.84 (948)	6.44 (3280)	5.63 (2106)
QSN $p_{re} = 0.3$	4.01 (575)	4.15 (405)	4.88 (610)	4.33 (9175)	5.11 (1150)	4.39 (4746)	4.48 (2777)
QSN $p_{re} = 0.4$	3.31 (688)	4.09 (497)	5.07 (630)	2.95 (13719)	4.44 (360)	2.96 (6409)	3.80 (3717)
QSN $p_{re} = 0.5$	3.02 (812)	3.47 (808)	4.81 (669)	2.71 (18487)	2.51 (10989)	2.04 (15709)	**3.09 (7912)**
QSN $p_{re} = 0.6$	3.47 (666)	4.10 (340)	3.95 (800)	3.48 (10868)	3.29 (5066)	2.74 (9241)	3.50 (4497)
QSN $p_{re} = 0.7$	4.13 (585)	5.03 (398)	4.69 (583)	4.22 (10462)	4.01 (1121)	4.31 (4593)	4.40 (2957)
QSN $p_{re} = 0.8$	5.33 (482)	6.36 (87)	4.82 (532)	5.70 (7831)	7.61 (634)	5.85 (3768)	5.95 (2222)
QSN $p_{re} = 0.9$	7.57 (150)	7.49 (85)	6.24 (321)	8.21 (3274)	6.79 (341)	8.31 (1731)	7.43 (984)
QSN $p_{re} = 1.0$	10.72 (2)	8.95 (117)	9.85 (47)	10.67 (13)	9.53 (13)	10.27 (9)	10.00 (34)

2.3 Network Size N=2000

Table S11. [$N = 2000$, $\langle k \rangle = 6.759$] Comparison of the original QSN and the QSN with re-directed edges in terms of exact controllability (EC).

EC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
QSN	10.16 (2)	7.74 (214)	7.66 (615)	10.08 (1)	9.24 (3)	9.34 (5)	9.04 (140)
QSN $p_{re} = 0.1$	8.80 (6)	6.56 (139)	7.56 (265)	8.50 (3)	7.33 (7)	7.81 (12)	7.76 (72)
QSN $p_{re} = 0.2$	6.30 (5)	5.69 (229)	6.94 (290)	6.51 (3)	5.40 (6)	5.79 (10)	6.10 (91)
QSN $p_{re} = 0.3$	4.89 (15)	3.80 (258)	4.90 (564)	4.50 (4)	3.62 (46)	3.93 (31)	4.27 (153)
QSN $p_{re} = 0.4$	2.38 (241)	4.71 (401)	5.70 (601)	2.33 (28)	2.69 (37)	2.61 (42)	3.41 (225)
QSN $p_{re} = 0.5$	1.55 (1363)	2.91 (1034)	4.10 (1057)	1.48 (184)	2.76 (96)	2.03 (144)	**2.47 (646)**
QSN $p_{re} = 0.6$	2.35 (420)	6.56 (40)	5.15 (550)	2.88 (17)	3.56 (32)	3.16 (35)	3.94 (182)
QSN $p_{re} = 0.7$	4.54 (13)	5.30 (117)	3.78 (560)	4.22 (19)	4.97 (3)	5.08 (11)	4.65 (121)
QSN $p_{re} = 0.8$	6.30 (38)	6.06 (91)	5.55 (299)	6.42 (2)	6.89 (23)	6.95 (5)	6.36 (76)
QSN $p_{re} = 0.9$	8.14 (1)	6.94 (199)	6.65 (262)	8.24 (2)	8.80 (2)	8.66 (4)	7.91 (78)
QSN $p_{re} = 1.0$	10.59 (1)	9.72 (22)	8.00 (315)	10.82 (1)	10.74 (1)	10.65 (1)	10.08 (57)

Table S12. $[N = 2000, \langle k \rangle = 6.759]$ Comparison of the original QSN and the QSN with re-directed edges in terms of structural controllability (SC).

SC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
QSN	10.16 (2)	7.74 (214)	7.64 (615)	10.08 (1)	9.24 (3)	9.34 (5)	9.03 (140)
QSN $p_{re} = 0.1$	8.80 (6)	6.53 (139)	7.55 (265)	8.50 (3)	7.33 (7)	7.81 (12)	7.76 (72)
QSN $p_{re} = 0.2$	6.29 (5)	5.70 (229)	6.94 (290)	6.50 (3)	5.40 (6)	5.80 (10)	6.10 (91)
QSN $p_{re} = 0.3$	4.88 (15)	3.80 (258)	4.90 (564)	4.50 (4)	3.61 (46)	3.93 (31)	4.27 (153)
QSN $p_{re} = 0.4$	2.38 (241)	4.72 (401)	5.71 (601)	2.33 (29)	2.69 (36)	2.61 (42)	3.41 (225)
QSN $p_{re} = 0.5$	1.55 (1362)	2.91 (1034)	4.10 (1057)	1.49 (183)	2.75 (96)	2.01 (149)	**2.47 (647)**
QSN $p_{re} = 0.6$	2.35 (420)	6.57 (40)	5.15 (550)	2.88 (17)	3.56 (29)	3.17 (35)	3.95 (182)
QSN $p_{re} = 0.7$	4.55 (13)	5.30 (117)	3.78 (560)	4.22 (19)	4.98 (3)	5.08 (11)	4.65 (121)
QSN $p_{re} = 0.8$	6.30 (38)	6.08 (91)	5.54 (299)	6.43 (2)	6.89 (23)	6.95 (5)	6.37 (76)
QSN $p_{re} = 0.9$	8.13 (1)	6.94 (199)	6.66 (262)	8.24 (2)	8.80 (2)	8.66 (4)	7.91 (78)
QSN $p_{re} = 1.0$	10.59 (1)	9.71 (22)	8.02 (315)	10.82 (1)	10.74 (1)	10.65 (1)	10.09 (57)

Table S13. $[N = 2000, \langle k \rangle = 10]$ Comparison of the original QSN and the QSN with re-directed edges in terms of exact controllability (EC).

EC	**R_N**	**TB_N**	**TD_N**	**R_E**	**TB_E**	**TD_E**	**Average**
QSN	10.69 (4)	7.62 (213)	8.40 (385)	10.08 (0)	9.24 (2)	9.69 (3)	9.29 (101)
QSN $p_{re} = 0.1$	8.29 (18)	8.05 (31)	7.68 (191)	8.50 (4)	7.56 (3)	8.06 (13)	8.02 (43)
QSN $p_{re} = 0.2$	6.25 (48)	7.16 (144)	6.15 (379)	6.37 (7)	4.97 (19)	5.97 (19)	6.14 (103)
QSN $p_{re} = 0.3$	4.07 (310)	5.83 (241)	4.33 (661)	4.62 (22)	3.45 (49)	4.28 (25)	4.43 (218)
QSN $p_{re} = 0.4$	2.29 (672)	4.41 (419)	3.85 (1259)	2.96 (45)	3.13 (32)	2.32 (77)	3.16 (417)
QSN $p_{re} = 0.5$	1.67 (1570)	4.08 (964)	5.29 (692)	1.58 (173)	2.34 (88)	2.36 (93)	**2.89 (597)**
QSN $p_{re} = 0.6$	3.41 (116)	3.45 (748)	5.06 (517)	2.38 (48)	3.92 (8)	3.00 (46)	3.54 (247)
QSN $p_{re} = 0.7$	4.51 (165)	4.13 (287)	4.18 (782)	4.06 (31)	5.20 (9)	4.78 (16)	4.47 (215)
QSN $p_{re} = 0.8$	6.34 (128)	5.55 (285)	5.54 (276)	6.42 (8)	6.73 (10)	6.55 (14)	6.19 (120)
QSN $p_{re} = 0.9$	8.37 (7)	6.64 (148)	6.34 (438)	8.27 (2)	8.75 (6)	8.37 (10)	7.79 (102)
QSN $p_{re} = 1.0$	10.12 (1)	9.08 (163)	9.17 (188)	10.76 (0)	10.73 (1)	10.64 (1)	10.08 (59)

Table S14. $[N = 2000, \langle k \rangle = 10]$ Comparison of the original QSN and the QSN with re-directed edges in terms of structural controllability (SC).

SC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
QSN	10.67 (4)	7.62 (206)	8.41 (385)	10.08 (0)	9.24 (2)	9.69 (3)	9.29 (100)
QSN $p_{re} = 0.1$	8.29 (18)	8.05 (31)	7.69 (191)	8.50 (4)	7.56 (3)	8.05 (13)	8.02 (43)
QSN $p_{re} = 0.2$	6.26 (48)	7.16 (144)	6.15 (379)	6.37 (7)	4.97 (18)	5.98 (19)	6.15 (103)
QSN $p_{re} = 0.3$	4.07 (310)	5.83 (241)	4.30 (661)	4.61 (22)	3.47 (46)	4.28 (25)	4.43 (218)
QSN $p_{re} = 0.4$	2.30 (672)	4.41 (420)	3.86 (1248)	2.95 (45)	3.12 (31)	2.32 (77)	3.16 (416)
QSN $p_{re} = 0.5$	1.67 (1571)	4.08 (968)	5.27 (696)	1.58 (173)	2.33 (88)	2.36 (94)	**2.88 (598)**
QSN $p_{re} = 0.6$	3.41 (115)	3.46 (742)	5.07 (517)	2.39 (48)	3.91 (8)	3.00 (47)	3.54 (246)
QSN $p_{re} = 0.7$	4.50 (163)	4.13 (287)	4.18 (782)	4.06 (31)	5.20 (9)	4.78 (16)	4.47 (215)
QSN $p_{re} = 0.8$	6.34 (128)	5.55 (285)	5.55 (276)	6.42 (8)	6.73 (9)	6.55 (14)	6.19 (120)
QSN $p_{re} = 0.9$	8.36 (7)	6.64 (158)	6.35 (438)	8.27 (2)	8.75 (6)	8.37 (10)	7.79 (104)
QSN $p_{re} = 1.0$	10.14 (1)	9.08 (162)	9.16 (188)	10.76 (0)	10.73 (1)	10.64 (1)	10.09 (59)

Table S15. $[N = 2000, \langle k \rangle = 20]$ Comparison of the original QSN and the QSN with re-directed edges in terms of exact controllability (EC).

EC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
QSN	10.09 (2)	8.71 (225)	8.91 (237)	10.11 (0)	9.63 (1)	6.92 (52)	9.06 (86)
QSN $p_{re} = 0.1$	7.88 (107)	7.43 (215)	7.63 (329)	8.33 (9)	7.05 (12)	9.59 (1)	7.99 (112)
QSN $p_{re} = 0.2$	5.40 (684)	5.55 (469)	5.59 (816)	6.11 (57)	5.54 (4)	6.56 (29)	5.79 (343)
QSN $p_{re} = 0.3$	3.86 (1077)	4.39 (581)	4.57 (1197)	4.27 (99)	3.83 (16)	4.77 (44)	4.28 (502)
QSN $p_{re} = 0.4$	3.15 (1178)	3.95 (1196)	4.12 (1338)	2.57 (176)	3.36 (40)	3.17 (51)	3.39 (663)
QSN $p_{re} = 0.5$	2.63 (1744)	3.87 (1228)	4.24 (1415)	2.77 (138)	2.34 (83)	2.06 (152)	**2.99 (793)**
QSN $p_{re} = 0.6$	2.84 (1316)	4.28 (1063)	5.15 (1145)	3.26 (108)	3.23 (20)	3.29 (46)	3.68 (616)
QSN $p_{re} = 0.7$	4.31 (999)	4.82 (686)	4.63 (1082)	4.15 (80)	4.50 (35)	4.69 (47)	4.52 (488)
QSN $p_{re} = 0.8$	6.23 (364)	6.11 (204)	4.27 (1113)	5.42 (67)	7.26 (3)	6.68 (34)	5.99 (298)
QSN $p_{re} = 0.9$	8.78 (69)	6.96 (266)	7.10 (429)	8.29 (10)	8.58 (10)	8.15 (19)	7.98 (134)
QSN $p_{re} = 1.0$	10.82 (1)	9.91 (15)	9.79 (102)	10.73 (1)	10.70 (1)	10.14 (1)	10.35 (20)

Table S16. $[N = 2000, \langle k \rangle = 20]$ Comparison of the original QSN and the QSN with re-directed edges in terms of structural controllability (SC).

SC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
QSN	10.10 (2)	8.69 (228)	8.91 (237)	10.12 (0)	9.64 (1)	6.89 (52)	9.06 (87)
QSN $p_{re} = 0.1$	7.87 (107)	7.43 (207)	7.63 (329)	8.34 (9)	7.05 (12)	9.59 0	7.98 (111)
QSN $p_{re} = 0.2$	5.41 (683)	5.56 (467)	5.59 (812)	6.10 (57)	5.55 (4)	6.56 (28)	5.80 (342)
QSN $p_{re} = 0.3$	3.85 (1077)	4.38 (598)	4.57 (1197)	4.27 (99)	3.85 (16)	4.76 (43)	4.28 (505)
QSN $p_{re} = 0.4$	3.15 (1180)	3.96 (1196)	4.12 (1336)	2.59 (174)	3.37 (40)	3.18 (50)	3.39 (663)
QSN $p_{re} = 0.5$	2.64 (1736)	3.88 (1228)	4.24 (1417)	2.75 (140)	2.38 (82)	2.07 (151)	**2.99 (792)**
QSN $p_{re} = 0.6$	2.84 (1323)	4.26 (1063)	5.14 (1145)	3.27 (108)	3.18 (22)	3.28 (45)	3.66 (618)
QSN $p_{re} = 0.7$	4.31 (999)	4.83 (686)	4.63 (1082)	4.14 (80)	4.50 (35)	4.71 (46)	4.52 (488)
QSN $p_{re} = 0.8$	6.23 (364)	6.12 (204)	4.27 (1113)	5.42 (67)	7.23 (3)	6.68 (33)	5.99 (297)
QSN $p_{re} = 0.9$	8.79 (69)	6.97 (264)	7.10 (429)	8.28 (10)	8.58 (10)	8.15 (18)	7.98 (133)
QSN $p_{re} = 1.0$	10.81 (1)	9.91 (15)	9.79 (102)	10.71 (1)	10.69 (1)	10.14 (0)	10.34 (20)

3 QSN with $p_{re} = 0.5$ vs. Other Network Topologies Comparison Tables

The comparisons of the overall best QSN variants (QSN with $p_{re} = 0.5$) and other network topologies are given in Tables. S17 to S32. The network size is set to $N = 500$, $N = 1000$, and $N = 2000$, respectively. For $N = 500$, the average degree of networks is set to $\langle k \rangle = 6.069$ and $\langle k \rangle = 10$, respectively. For $N = 1000$ and $N = 2000$, the average degree is set to $\langle k \rangle = 6.069$, $\langle k \rangle = 10$, and $\langle k \rangle = 20$, respectively. When $\langle k \rangle = 6.069$, QSN with $p_{re} = 0.5$ is compared to random graph (RG), multiplex congruence network (MCN), random triangle network (RTN), and random rectangle network (RRN). When $\langle k \rangle = 10$ and $\langle k \rangle = 20$, QSN with $p_{re} = 0.5$ is compared to RG, RTN, and RRN only. MCN is excluded because the average degree of an MCN cannot be set manually.

In each cell of the table, the real number represents the average rank, and the integer inside the parentheses mean the number of winning times. Bold real numbers (with gray-shaded) represent the minimum average rank, and bold numbers inside parentheses (with gray-shaded) mean the maximum average number of winning times.

3.1 Network Size N=500

Table S17. $[N = 500, \langle k \rangle = 5.38]$ Comparison of robustness of exact controllability (EC) among 5 networks.

EC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
RG	3.07 (7)	2.38 (80)	2.69 (202)	2.54 (3)	3.72 (67)	3.36 (370)	2.96 (122)
RTN	3.73 (2)	3.75 (12)	4.49 (91)	3.84 (64)	1.97 (868)	2.55 (395)	3.39 (239)
RRN	1.95 (140)	2.34 (177)	3.45 (148)	2.76 (160)	1.61 (1481)	1.38 (2167)	2.25 (712)
MCN	4.96 (4)	4.95 (6)	2.36 (267)	4.81 (2)	5.00 (2)	4.99 (5)	4.51 (48)
QSN $p_{re} = 0.5$	1.30 (361)	1.58 (306)	2.01 (263)	1.05 (2654)	2.71 (477)	2.72 (429)	**1.89 (748)**

Table S18. $[N = 500, \langle k \rangle = 5.38]$ Comparison of robustness of structural controllability (SC) among 5 networks.

SC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
RG	3.08 (7)	2.38 (80)	2.69 (202)	2.55 (3)	3.70 (67)	3.36 (370)	2.96 (122)
RTN	3.72 (2)	3.75 (12)	4.49 (91)	3.83 (73)	2.00 (868)	2.55 (395)	3.39 (240)
RRN	1.96 (136)	2.35 (170)	3.45 (148)	2.76 (160)	1.57 (1500)	1.38 (2167)	2.24 (714)
MCN	4.96 (4)	4.95 (6)	2.38 (263)	4.80 (2)	5.00 (2)	4.99 (5)	4.51 (47)
QSN $p_{re} = 0.5$	1.29 (367)	1.57 (307)	2.00 (264)	1.05 (2654)	2.73 (426)	2.72 (429)	**1.89 (741)**

Table S19. $[N = 500, \langle k \rangle = 10]$ Comparison of robustness of exact controllability (EC) among 5 networks.

EC	**R_N**	**TB_N**	**TD_N**	**R_E**	**TB_E**	**TD_E**	**Average**
RG	2.43 (157)	1.85 (312)	2.23 (283)	2.12 (393)	3.29 (746)	3.46 (574)	2.56 (411)
RTN	3.31 (145)	3.52 (101)	3.46 (177)	3.78 (347)	2.57 (681)	2.54 (1074)	3.20 (421)
RRN	2.61 (176)	2.30 (203)	2.58 (237)	2.99 (345)	1.83 (2620)	1.42 (4376)	2.29 (1326)
QSN $p_{re} = 0.5$	1.65 (436)	2.32 (191)	1.73 (477)	1.11 (4998)	2.31 (1553)	2.58 (1290)	1.95 (1491)

Table S20. $[N = 500, \langle k \rangle = 10]$ Comparison of robustness of structural controllability (SC) among 5 networks.

SC	**R_N**	**TB_N**	**TD_N**	**R_E**	**TB_E**	**TD_E**	**Average**
RG	2.41 (164)	1.87 (312)	2.23 (283)	2.12 (393)	3.29 (746)	3.46 (574)	2.56 (412)
RTN	3.31 (143)	3.52 (101)	3.46 (177)	3.78 (347)	2.58 (642)	2.54 (1074)	3.20 (414)
RRN	2.61 (176)	2.29 (204)	2.58 (237)	2.99 (345)	1.83 (2611)	1.42 (4376)	2.29 (1325)
QSN $p_{re} = 0.5$	1.67 (429)	2.31 (189)	1.73 (477)	1.11 (4998)	2.30 (1539)	2.58 (1290)	**1.95 (1487)**

3.2 Network Size N=1000

Table S21. $[N = 1000, \langle k \rangle = 6.069]$ Comparison of robustness of exact controllability (EC) among 5 networks.

EC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
RG	3.02 (57)	2.71 (66)	2.95 (244)	2.34 (1)	3.28 (683)	3.54 (8)	2.97 (177)
RTN	3.50 (31)	3.25 (80)	4.40 (174)	4.04 (63)	2.28 (1237)	2.50 (1481)	3.33 (511)
RRN	2.29 (73)	2.84 (96)	2.98 (295)	2.77 (296)	1.31 (4494)	1.22 (5424)	2.24 **(1780)**
MCN	4.98 (4)	4.96 (4)	3.02 (284)	4.81 (5)	5.00 (7)	5.00 (4)	4.63 (51)
QSN $p_{re} = 0.5$	1.21 (878)	1.24 (901)	1.65 (800)	1.03 (6051)	3.14 (77)	2.74 (470)	**1.83** (1530)

Table S22. $[N = 1000, \langle k \rangle = 6.069]$ Comparison of robustness of structural controllability (SC) among 5 networks.

SC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
RG	3.02 (56)	2.72 (66)	2.95 (244)	2.34 (1)	3.28 (651)	3.55 (8)	2.98 (171)
RTN	3.50 (31)	3.24 (80)	4.40 (174)	4.05 (63)	2.27 (1237)	2.50 (1481)	3.33 (511)
RRN	2.30 (68)	2.84 (96)	2.98 (295)	2.78 (296)	1.31 (4511)	1.22 (5424)	2.24 **(1782)**
MCN	4.98 (4)	4.96 (4)	3.02 (284)	4.80 (5)	5.00 (7)	5.00 (4)	4.63 (51)
QSN $p_{re} = 0.5$	1.20 (883)	1.24 (901)	1.65 (798)	1.03 (6051)	3.14 (77)	2.73 (470)	**1.83** (1530)

Table S23. $[N = 1000, \langle k \rangle = 10]$ Comparison of robustness of exact controllability (EC) among 4 networks.

EC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
RG	1.96 (554)	1.72 (624)	1.93 (537)	3.90 (636)	3.00 (1962)	3.19 (2530)	2.62 (1141)
RTN	3.52 (192)	3.69 (104)	3.41 (275)	2.73 (1994)	2.65 (458)	2.39 (2996)	3.07 (1003)
RRN	2.27 (391)	2.61 (317)	2.98 (280)	1.67 (6073)	1.40 (7434)	2.03 (4744)	**2.16 (3207)**
QSN $p_{re} = 0.5$	2.25 (461)	1.99 (470)	1.67 (840)	1.70 (4990)	2.96 (436)	2.39 (3912)	**2.16** (1852)

Table S24. $[N = 1000, \langle k \rangle = 10]$ Comparison of robustness of structural controllability (SC) among 4 networks.

SC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
RG	1.97 (551)	1.72 (625)	1.93 (537)	3.90 (636)	3.00 (1958)	3.19 (2530)	2.62 (1140)
RTN	3.53 (192)	3.69 (104)	3.41 (275)	2.73 (1994)	2.64 (482)	2.40 (2996)	3.07 (1007)
RRN	2.25 (398)	2.60 (319)	2.98 (280)	1.67 (6073)	1.40 (7471)	2.03 (4744)	**2.16 (3214)**
QSN $p_{re} = 0.5$	2.26 (454)	1.99 (466)	1.67 (840)	1.70 (4990)	2.96 (486)	2.38 (3912)	**2.16** (1858)

Table S25. $[N = 1000, \langle k \rangle = 20]$ Comparison of robustness of exact controllability (EC) among 4 networks.

EC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
RG	2.48 (727)	2.10 (649)	1.90 (999)	2.24 (1192)	3.67 (199)	2.72 (7177)	2.52 (1824)
RTN	2.92 (631)	3.04 (445)	3.09 (565)	3.91 (1171)	2.61 (1688)	2.07 (7479)	2.94 (1997)
RRN	2.42 (713)	2.96 (352)	2.73 (597)	2.75 (1261)	1.54 (11622)	2.18 (11641)	2.43 (4364)
QSN $p_{re} = 0.5$	2.19 (797)	1.91 (798)	2.27 (640)	1.09 (19998)	2.17 (8566)	3.03 (6304)	**2.11 (6184)**

Table S26. $[N = 1000, \langle k \rangle = 20]$ Comparison of robustness of structural controllability (SC) among 4 networks.

SC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
RG	2.47 (726)	2.10 (648)	1.90 (999)	2.24 (1192)	3.68 (146)	2.72 (7177)	2.52 (1815)
RTN	2.93 (631)	3.04 (445)	3.09 (565)	3.91 (1171)	2.61 (1780)	2.07 (7499)	2.94 (2015)
RRN	2.41 (708)	2.96 (352)	2.73 (597)	2.75 (1261)	1.52 (12254)	2.18 (11641)	2.43 (4469)
QSN $p_{re} = 0.5$	2.18 (803)	1.91 (801)	2.27 (640)	1.09 (19998)	2.19 (8421)	3.03 (6304)	**2.11 (6161)**

3.3 Network Size N=2000

Table S27. $[N = 2000, \langle k \rangle = 6.759]$ Comparison of robustness of exact controllability (EC) among 5 networks.

EC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
RG	3.21 (57)	2.41 (321)	2.61 (304)	3.32 (1)	3.89 (1)	3.77 (1)	3.20 (114)
RTN	3.67 (34)	3.55 (51)	4.28 (304)	3.60 (8)	1.89 (49)	1.79 (64)	3.13 (85)
RRN	1.78 (601)	2.69 (456)	3.14 (462)	1.93 (27)	1.22 (179)	1.25 (186)	2.00 (319)
MCN	4.99 (1)	4.97 (19)	3.61 (363)	4.99 (1)	4.99 (1)	4.99 (1)	4.76 (64)
QSN $p_{re} = 0.5$	1.36 (1409)	1.38 (1456)	1.37 (1959)	1.16 (208)	3.01 (2)	3.20 (7)	**1.91 (840)**

Table S28. $[N = 2000, \langle k \rangle = 6.759]$ Comparison of robustness of structural controllability (SC) among 5 networks.

SC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
RG	3.21 (56)	2.41 (321)	2.60 (304)	3.32 (1)	3.89 (1)	3.77 (1)	3.20 (114)
RTN	3.67 (34)	3.55 (51)	4.28 (304)	3.60 (8)	1.89 (49)	1.79 (64)	3.13 (85)
RRN	1.76 (629)	2.69 (456)	3.14 (462)	1.93 (27)	1.22 (179)	1.25 (186)	2.00 (323)
MCN	4.99 (1)	4.97 (19)	3.61 (363)	4.99 (1)	4.99 (1)	4.99 (1)	4.76 (64)
QSN $p_{re} = 0.5$	1.37 (1384)	1.38 (1456)	1.37 (1959)	1.16 (208)	3.01 (2)	3.20 (7)	**1.92 (836)**

Table S29. $[N = 2000, \langle k \rangle = 10]$ Comparison of robustness of exact controllability (EC) among 4 networks.

EC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
RG	3.12 (130)	2.28 (494)	1.52 (1761)	2.35 (64)	3.68 (1)	3.50 (35)	2.74 (414)
RTN	3.64 (266)	3.58 (168)	3.51 (419)	3.85 (17)	2.02 (39)	1.76 (92)	3.06 (167)
RRN	1.64 (1124)	2.69 (632)	2.88 (510)	2.42 (24)	1.19 (168)	1.55 (119)	**2.06** (430)
QSN $p_{re} = 0.5$	1.59 (1195)	1.46 (1583)	2.08 (923)	1.38 (175)	3.11 (2)	3.20 (8)	2.14 **(648)**

Table S30. $[N = 2000, \langle k \rangle = 10]$ Comparison of robustness of structural controllability (SC) among 4 networks.

SC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
RG	3.12 (130)	2.28 (494)	1.53 (1756)	2.35 (64)	3.68 (1)	3.50 (35)	2.74 (413)
RTN	3.65 (266)	3.58 (168)	3.51 (419)	3.85 (17)	2.01 (39)	1.76 (92)	3.06 (167)
RRN	1.65 (1117)	2.69 (632)	2.88 (510)	2.42 (24)	1.19 (168)	1.55 (119)	**2.06** (428)
QSN $p_{re} = 0.5$	1.59 (1204)	1.46 (1583)	2.08 (947)	1.38 (175)	3.12 (2)	3.20 (8)	2.14 **(653)**

Table S31. $[N = 2000, \langle k \rangle = 20]$ Comparison of robustness of exact controllability (EC) among 4 networks.

EC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
RG	2.65 (1055)	1.97 (1542)	2.14 (1351)	2.00 (168)	3.37 (2)	2.88 (51)	2.50 (695)
RTN	2.85 (1198)	3.22 (766)	3.46 (719)	3.27 (98)	2.27 (9)	2.05 (77)	2.85 (478)
RRN	2.37 (1406)	2.46 (1052)	2.60 (1169)	2.17 (146)	1.05 (194)	2.01 (112)	**2.11** (680)
QSN $p_{re} = 0.5$	2.12 (1497)	2.36 (1179)	1.81 (1987)	2.57 (120)	3.32 (2)	3.06 (64)	2.54 **(808)**

Table S32. $[N = 2000, \langle k \rangle = 20]$ Comparison of robustness of structural controllability (SC) among 4 networks.

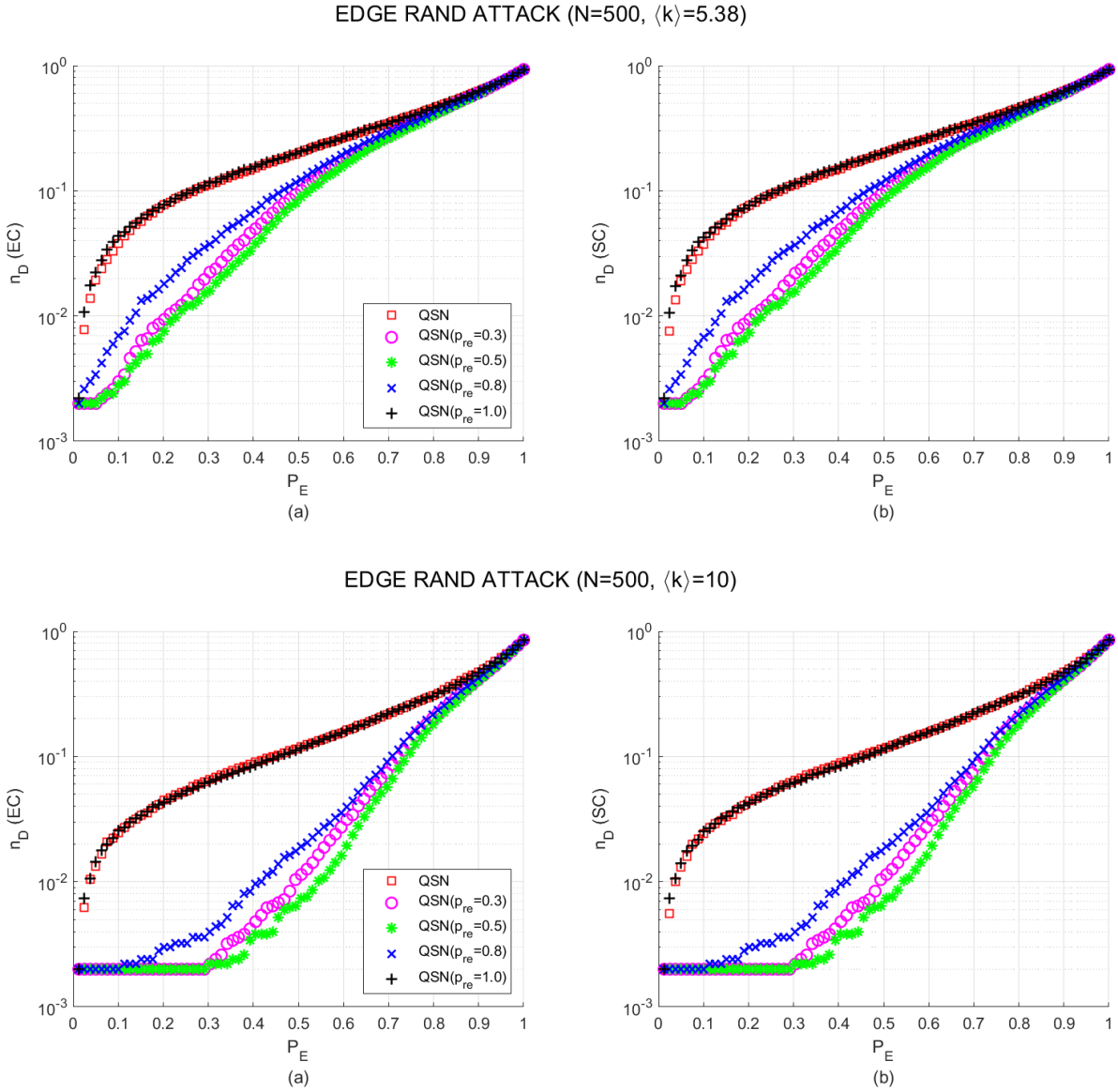
SC	R_N	TB_N	TD_N	R_E	TB_E	TD_E	Average
RG	2.66 (1052)	1.97 (1532)	2.14 (1351)	2.00 (168)	3.36 (2)	2.88 (51)	2.50 (693)
RTN	2.86 (1197)	3.22 (766)	3.46 (719)	3.27 (98)	2.26 (11)	2.05 (77)	2.85 (478)
RRN	2.36 (1407)	2.46 (1052)	2.60 (1169)	2.16 (147)	1.05 (194)	2.01 (112)	**2.11** (680)
QSN $p_{re} = 0.5$	2.12 (1499)	2.36 (1179)	1.81 (1987)	2.57 (120)	3.33 (2)	3.06 (64)	2.54 **(809)**

4 QSN vs. QSN Variants Comparison Curves

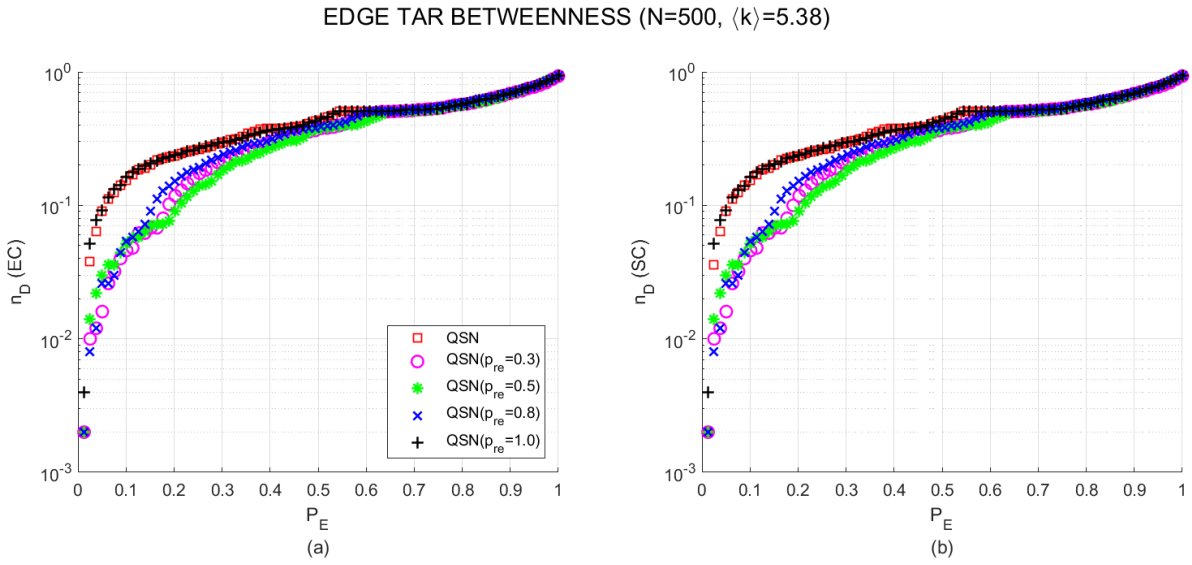
The comparison curves of controllability robustness are shown in the following figures. The left subplot (a) shows the results of exact controllability (EC); and the right subplot (b) shows the results of structural controllability (SC).

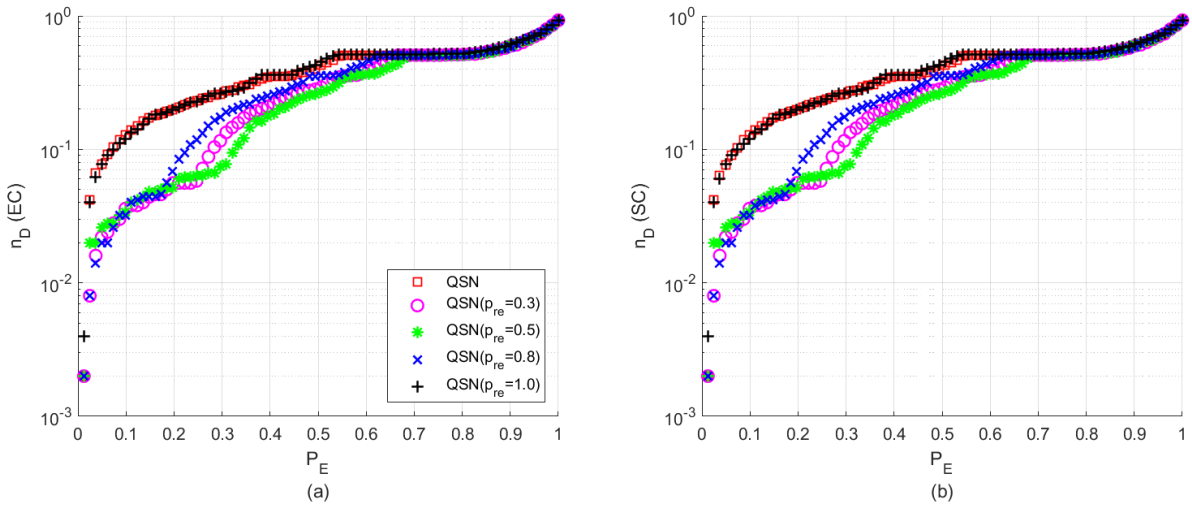
4.1 Network Size N=500

4.1.1 Edge Random Attack

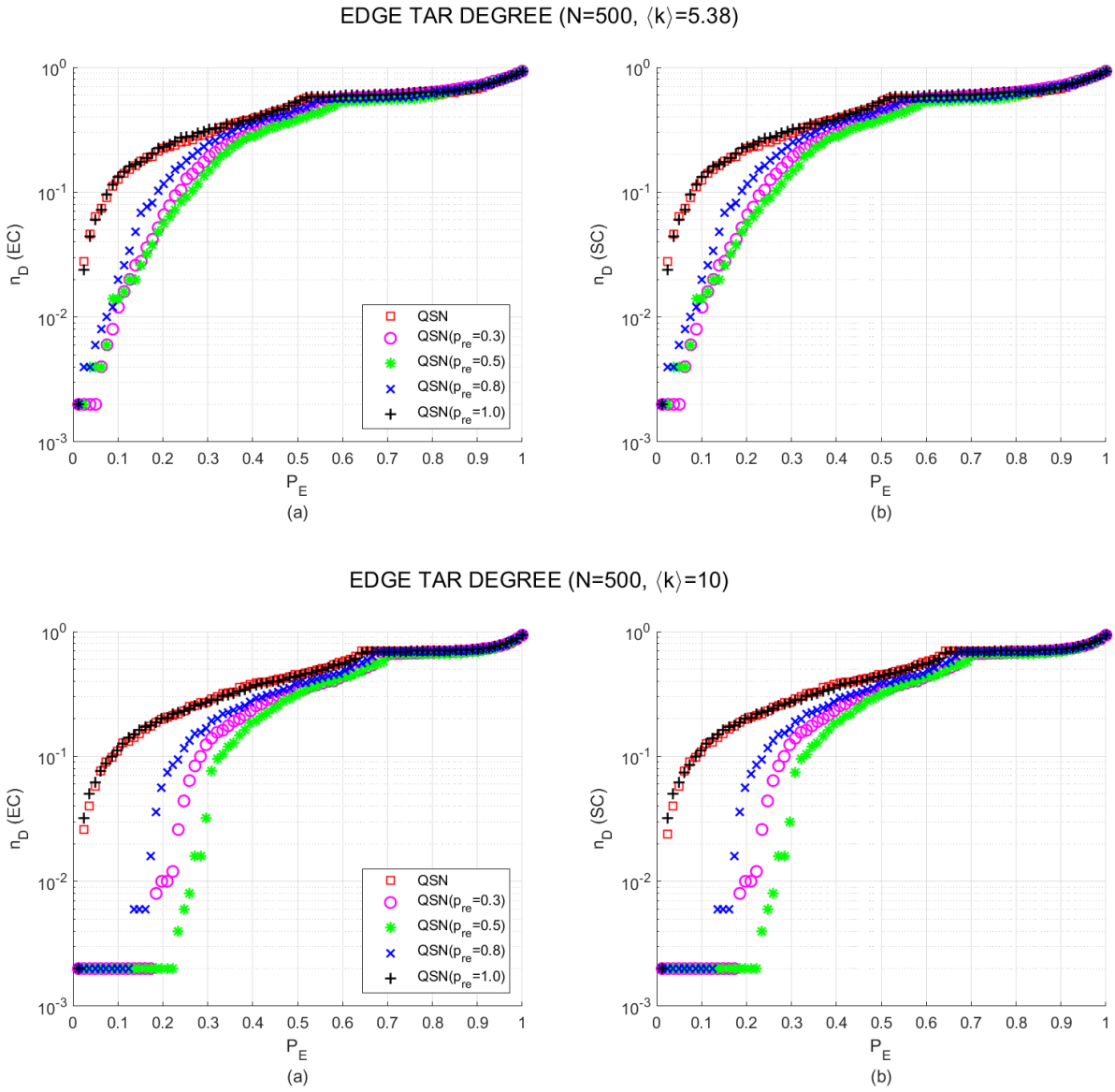


4.1.2 Edge Intentional (Betweenness-based) Attack

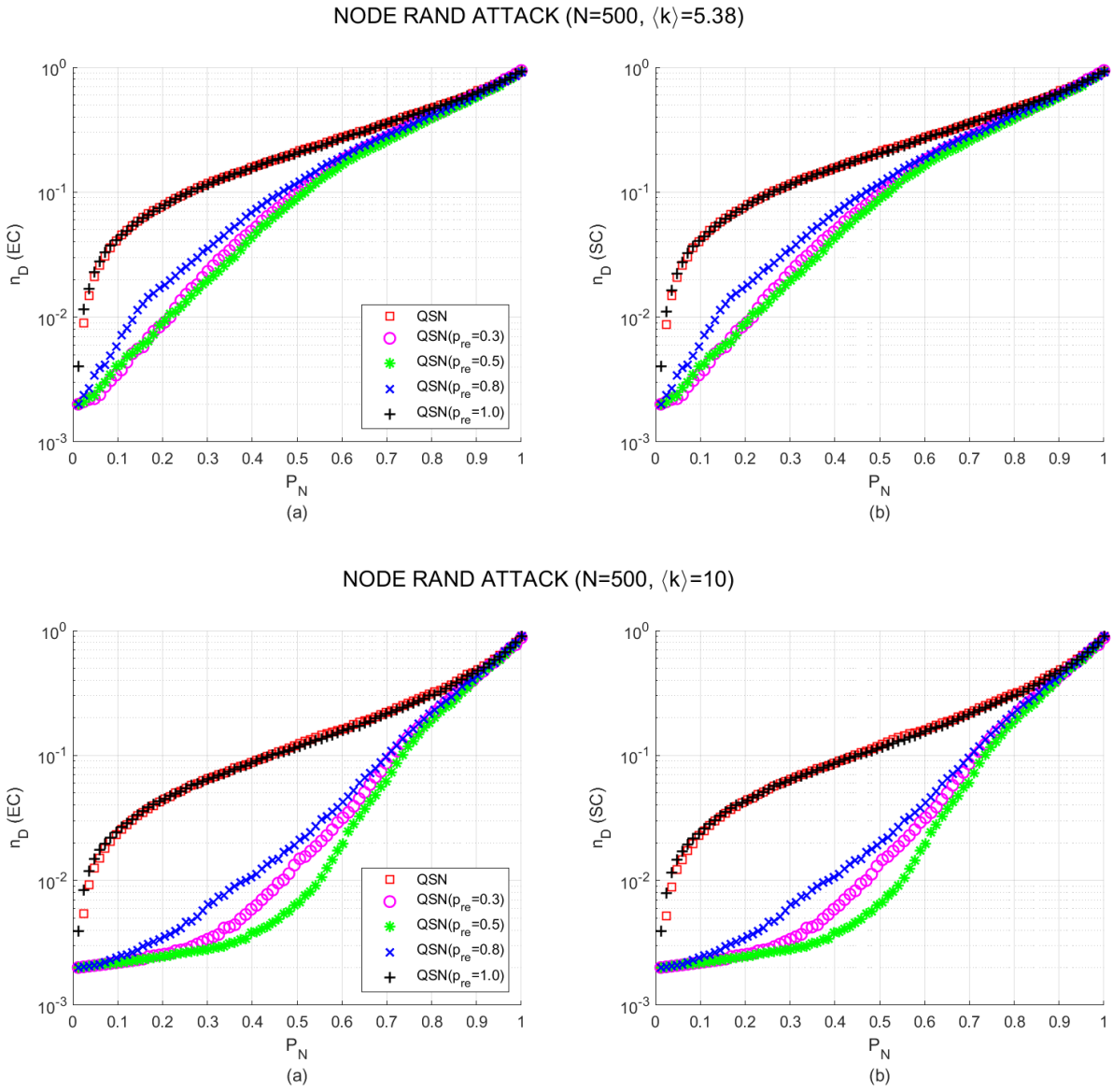




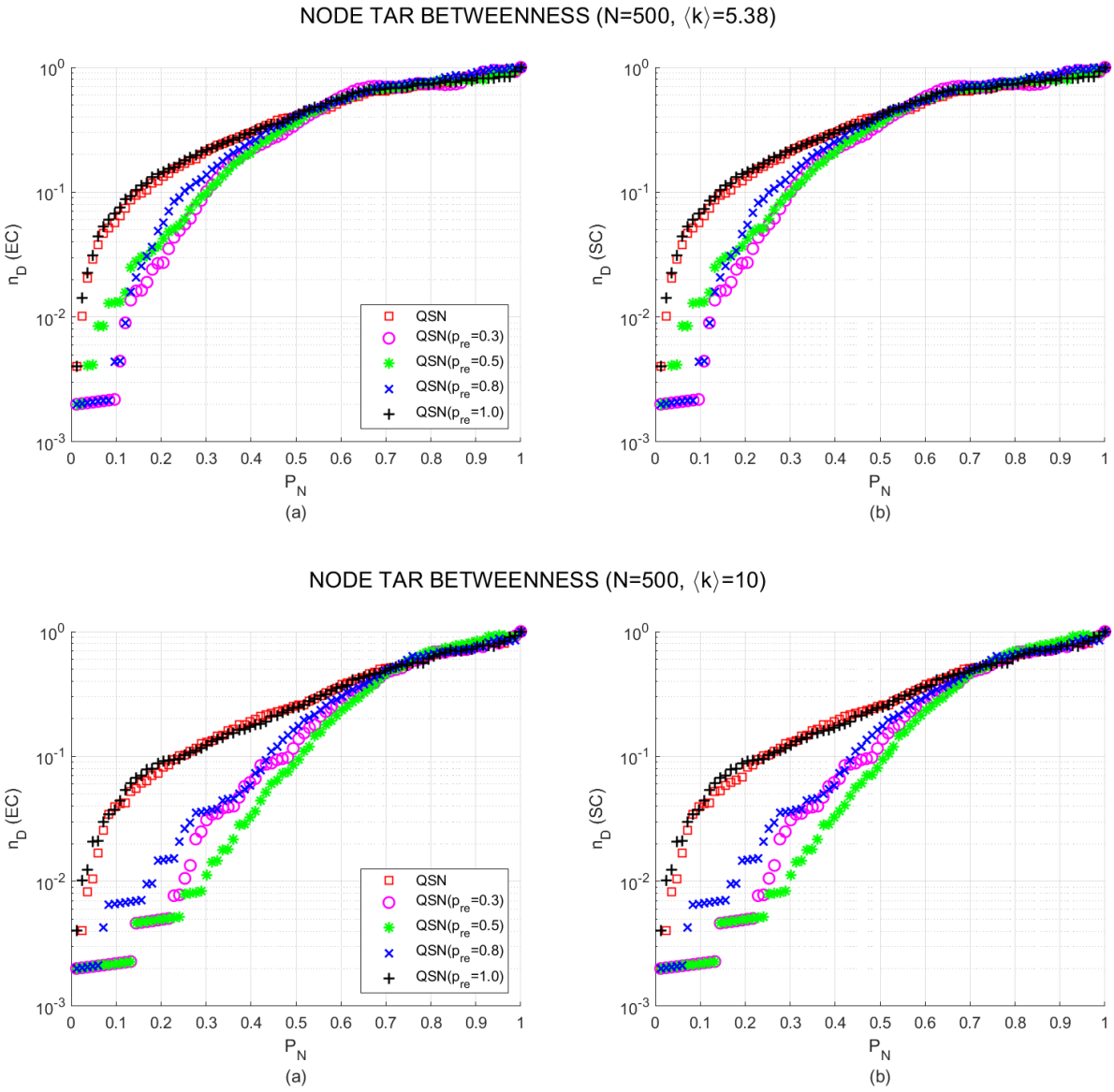
4.1.3 Edge Intentional (Degree-based) Attack



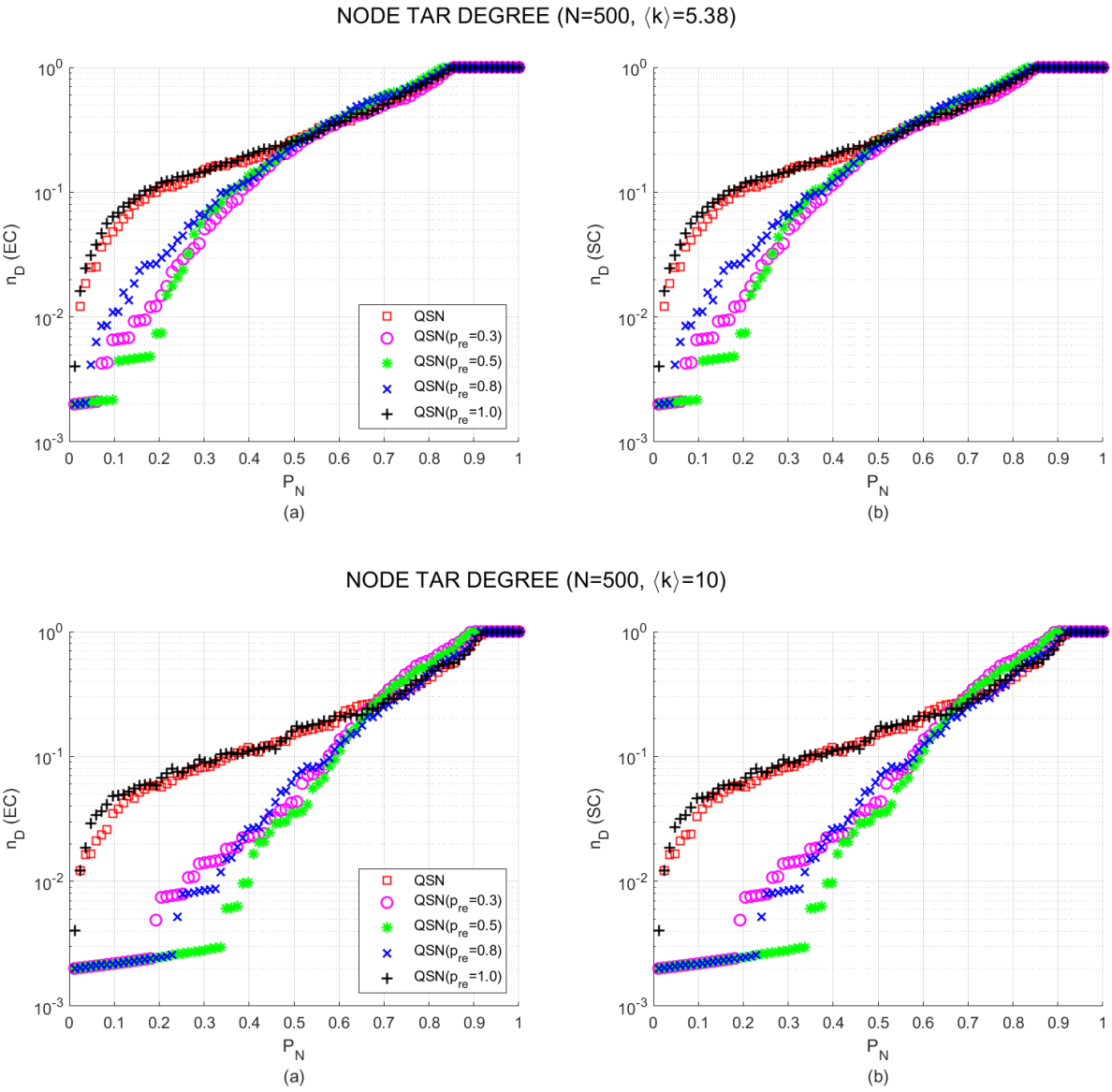
4.1.4 *Node Random Attack*



4.1.5 Node Intentional (Betweenness-based) Attack

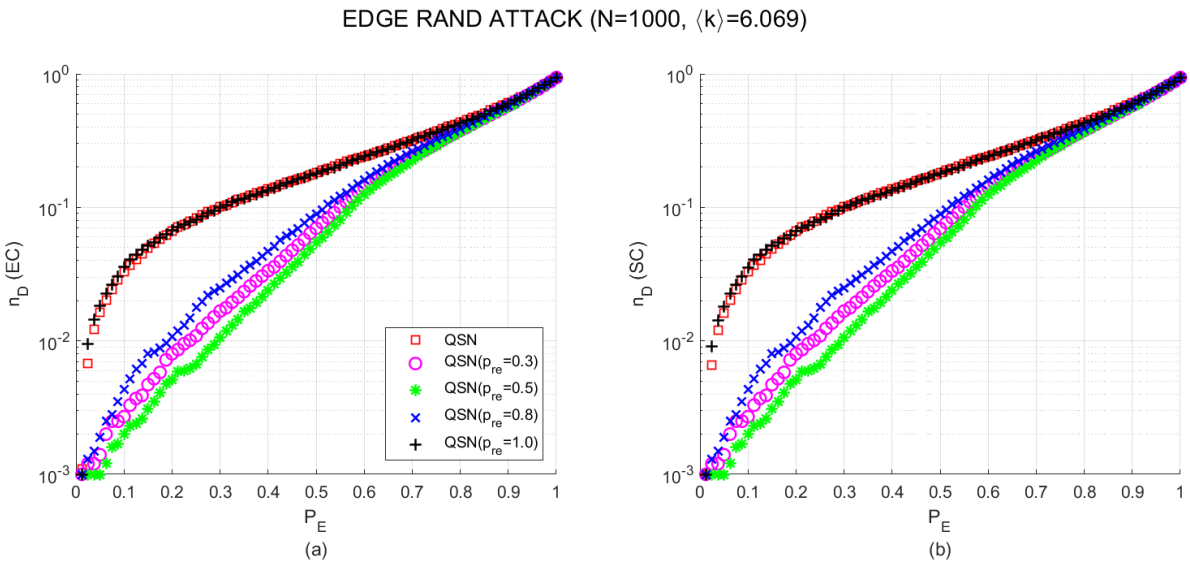


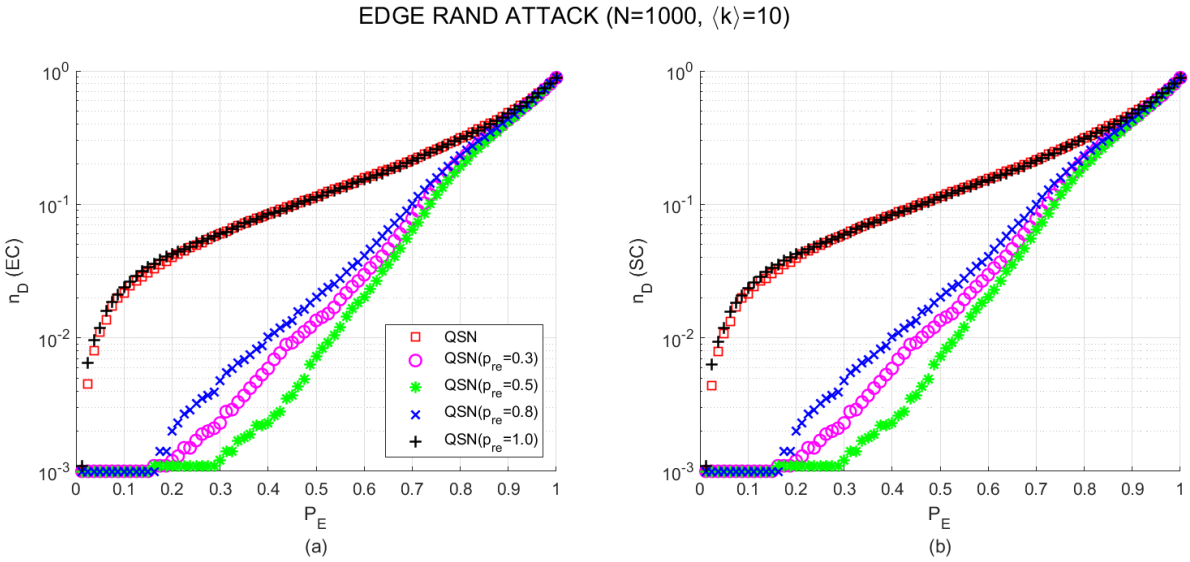
4.1.6 Node Intentional (Degree-based) Attack



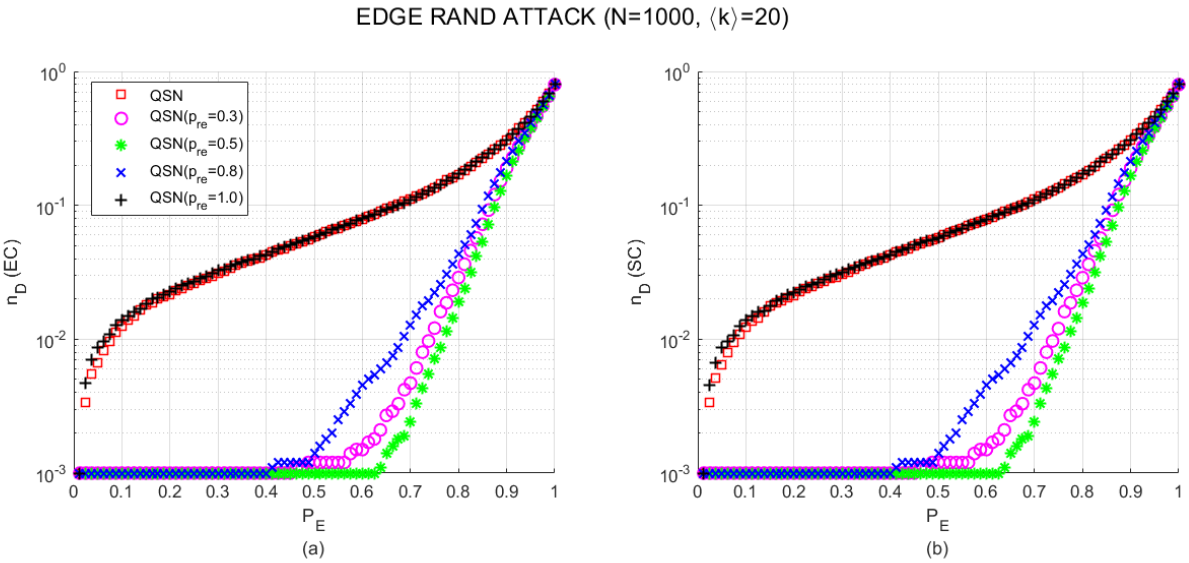
4.2 Network Size N=1000

4.2.1 Edge Random Attack

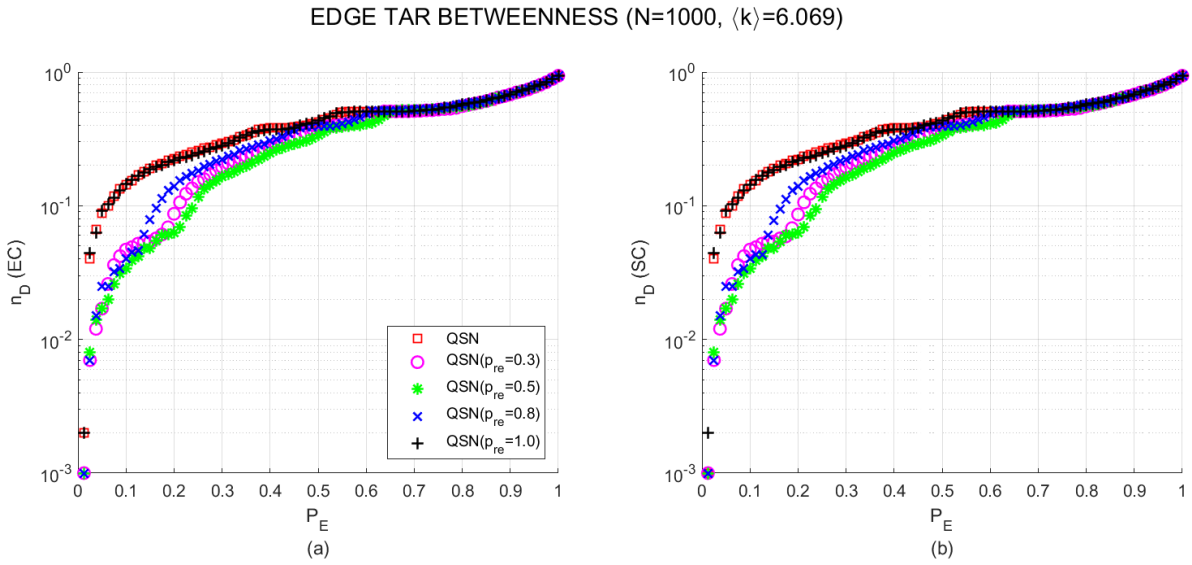


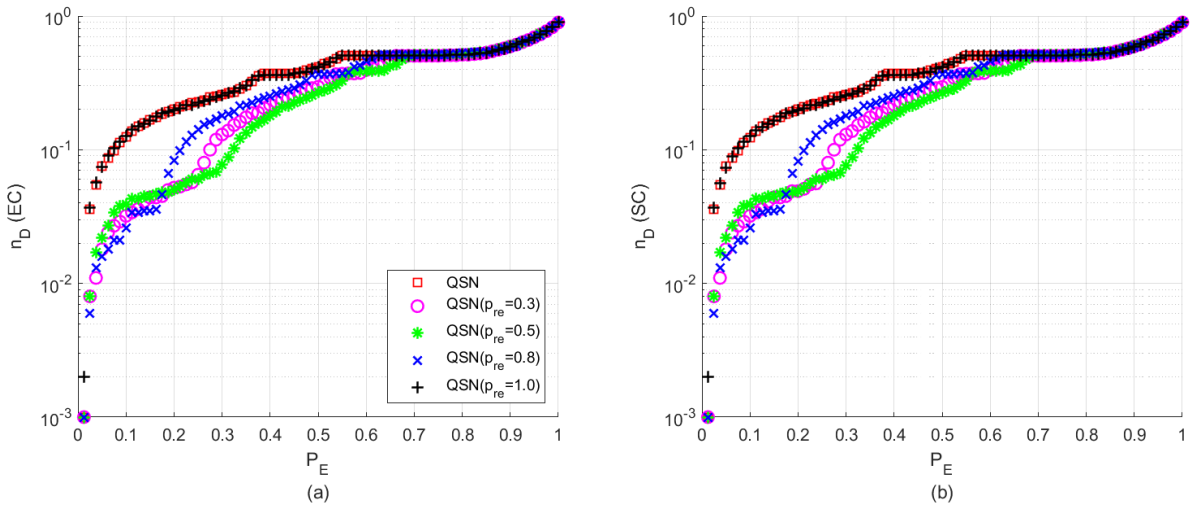


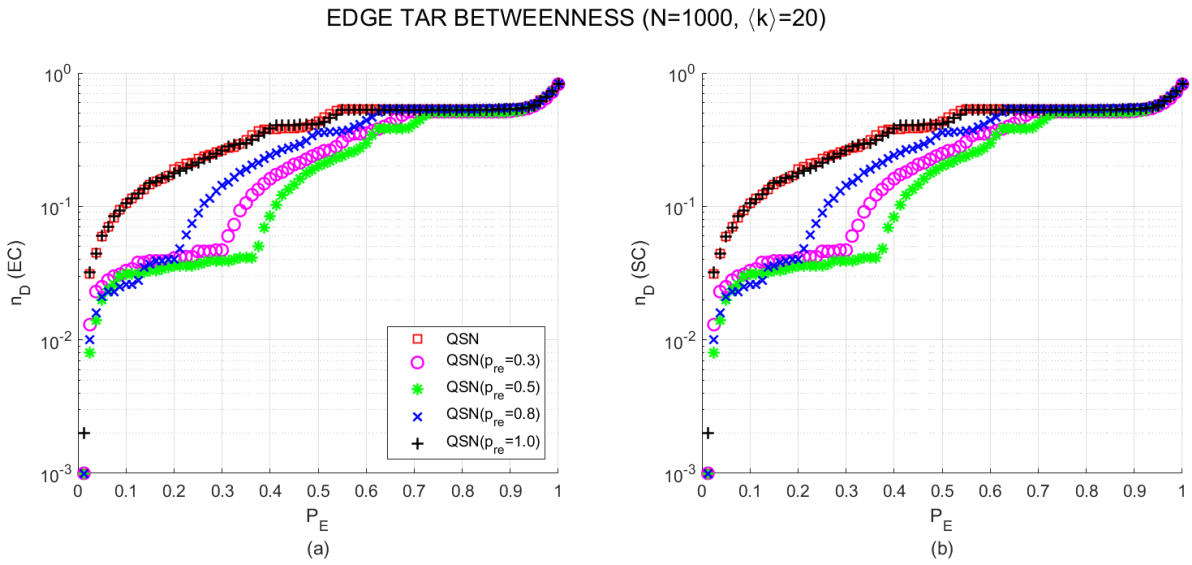
k


EDGE RAND ATTACK (N=1000, ⟨k⟩=20)
QSN
QSN(p_re=0.3)
QSN(p_re=0.5)
QSN(p_re=0.8)
QSN(p_re=1.0)
n_D (EC)
n_D (SC)
P_E
(a)
(b)

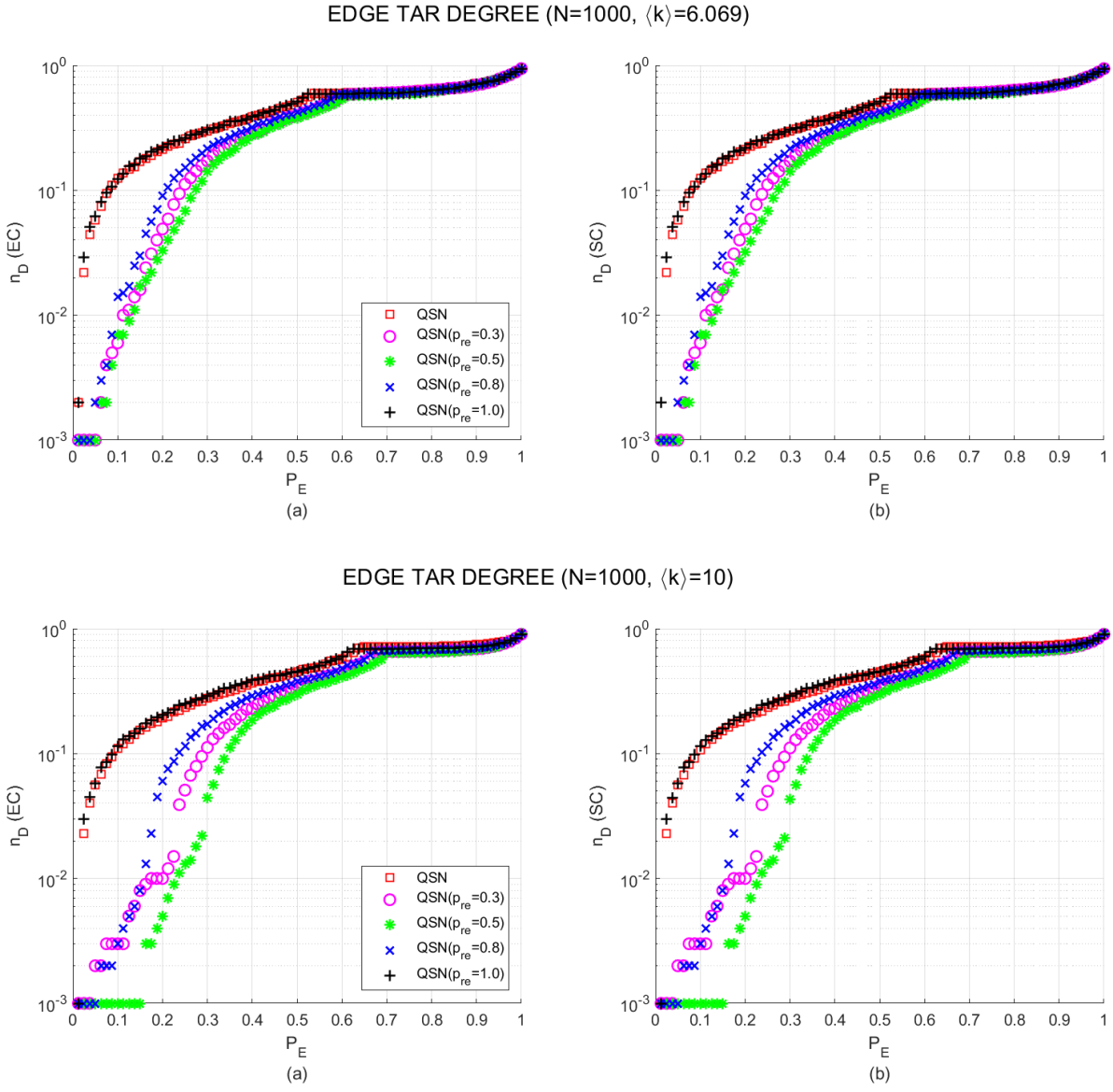
4.2.2 *Edge Intentional (Betweenness-based) Attack*



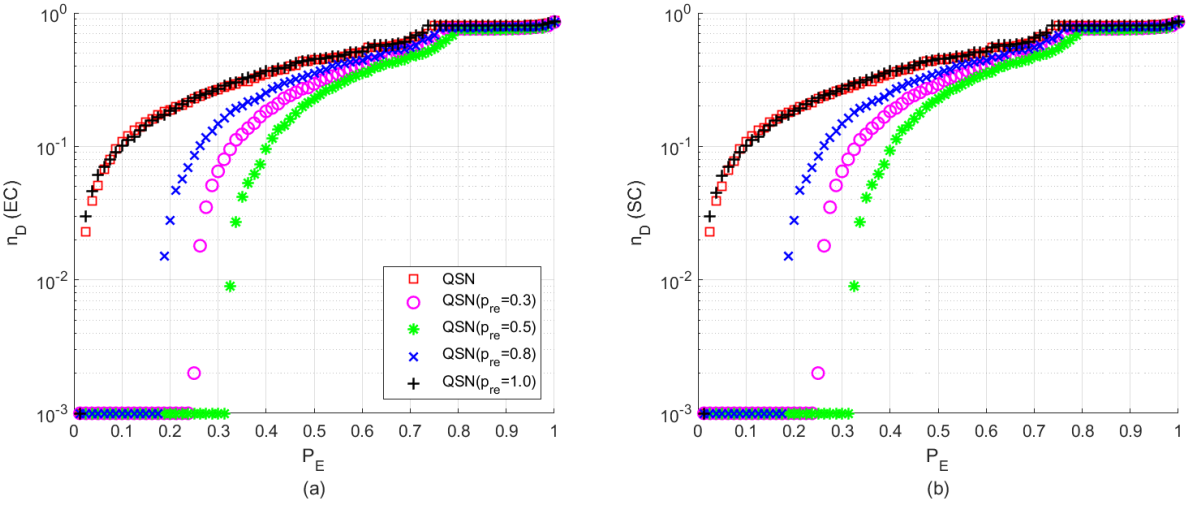



EDGE TAR BETWEENNESS (N=1000, ⟨k⟩=20)
n_D (EC)
P_E
(a)
QSN
QSN(p_{re}=0.3)
QSN(p_{re}=0.5)
QSN(p_{re}=0.8)
QSN(p_{re}=1.0)
n_D (SC)
P_E
(b)

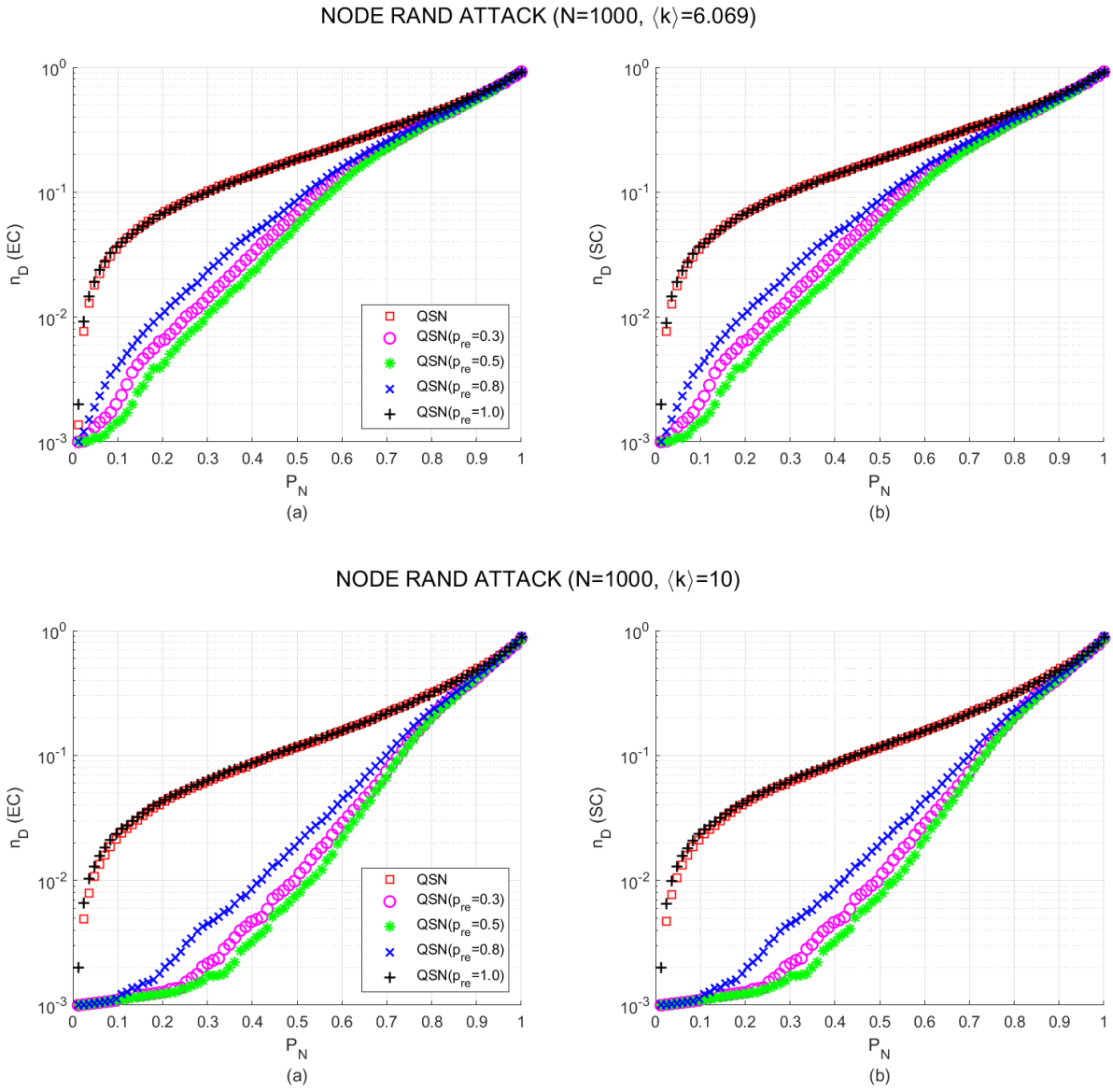
4.2.3 *Edge Intentional (Degree-based) Attack*

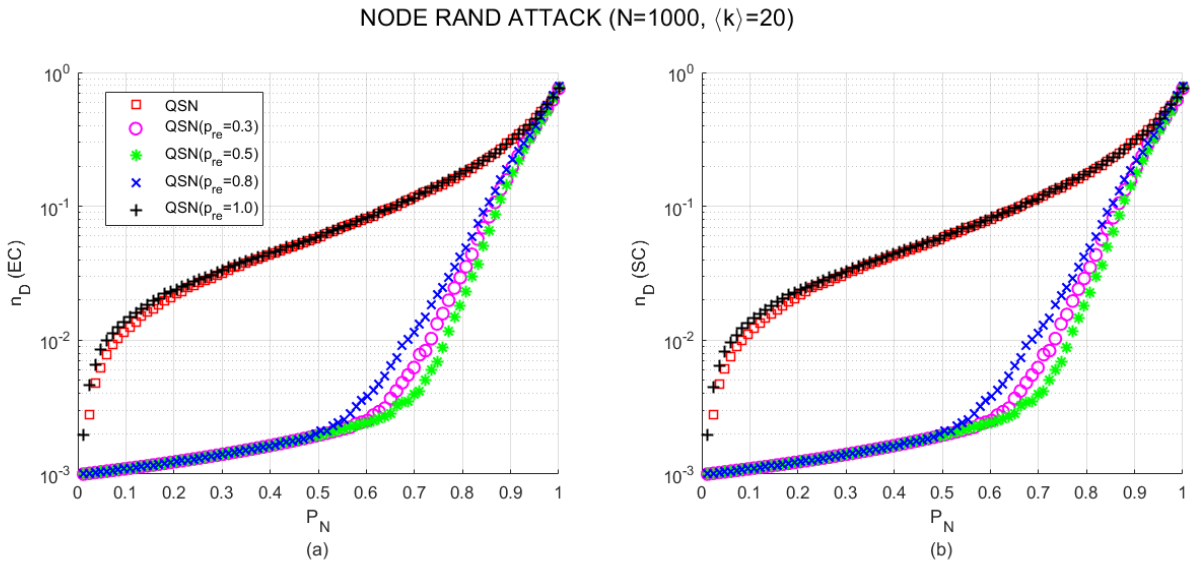


EDGE TAR DEGREE (N=1000, ⟨k⟩=20)

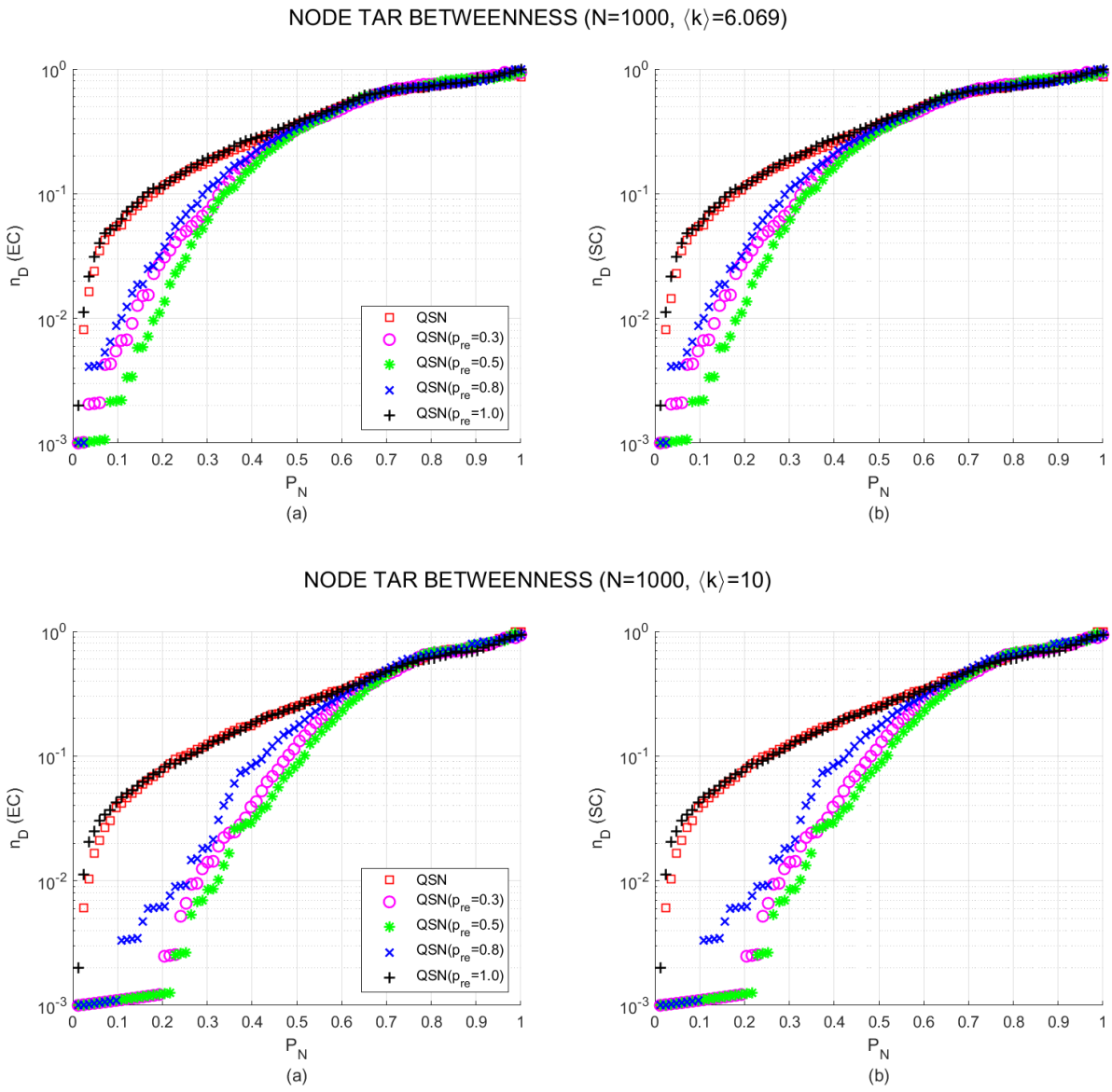


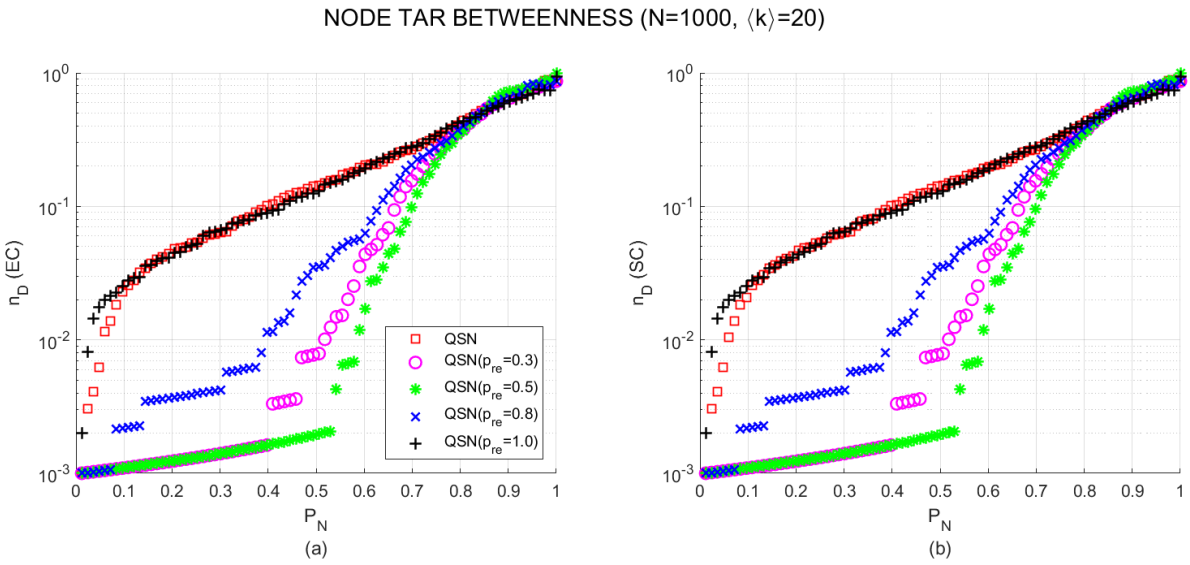
4.2.4 *Node Random Attack*



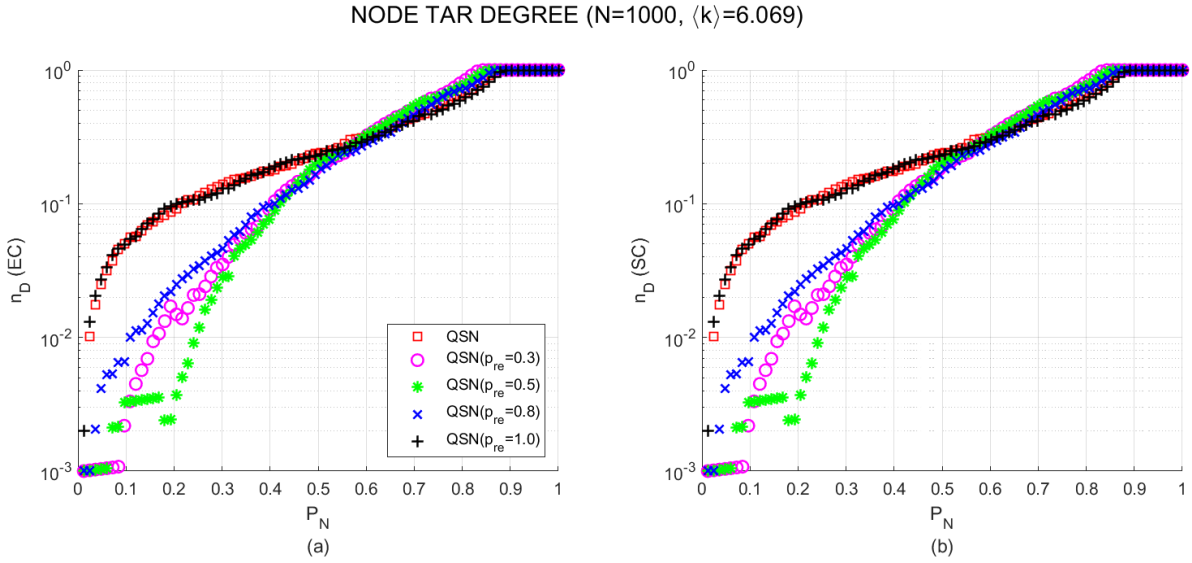

NODE RAND ATTACK (N=1000, ⟨k⟩=20)
QSN
QSN(p_re=0.3)
QSN(p_re=0.5)
QSN(p_re=0.8)
QSN(p_re=1.0)
n_D (EC)
n_D (SC)
P_N
(a)
(b)

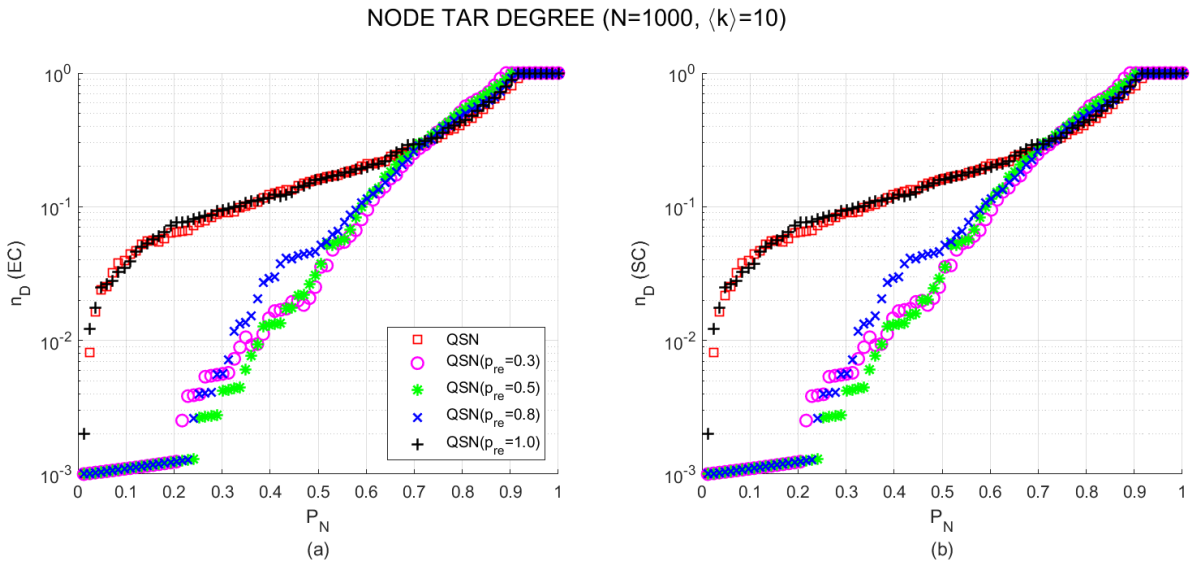
4.2.5 *Node Intentional (Betweenness-based) Attack*

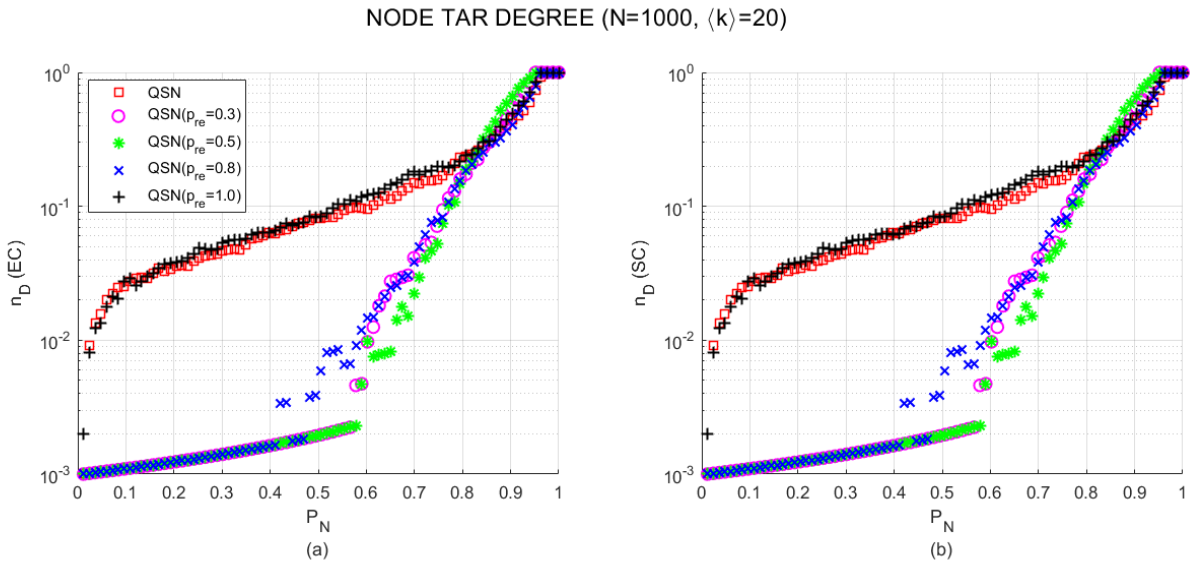



NODE TAR BETWEENNESS (N=1000, ⟨k⟩=20)
n_D (EC)
P_N
(a)
QSN
QSN(p_{re}=0.3)
QSN(p_{re}=0.5)
QSN(p_{re}=0.8)
QSN(p_{re}=1.0)
n_D (SC)
P_N
(b)

4.2.6 *Node Intentional (Degree-based) Attack*

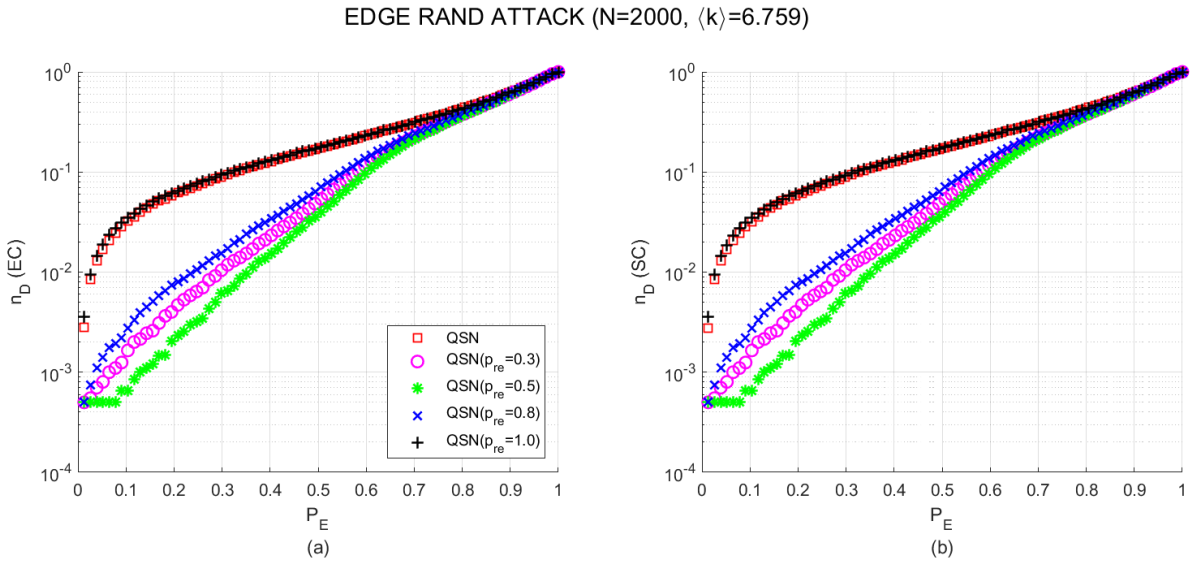


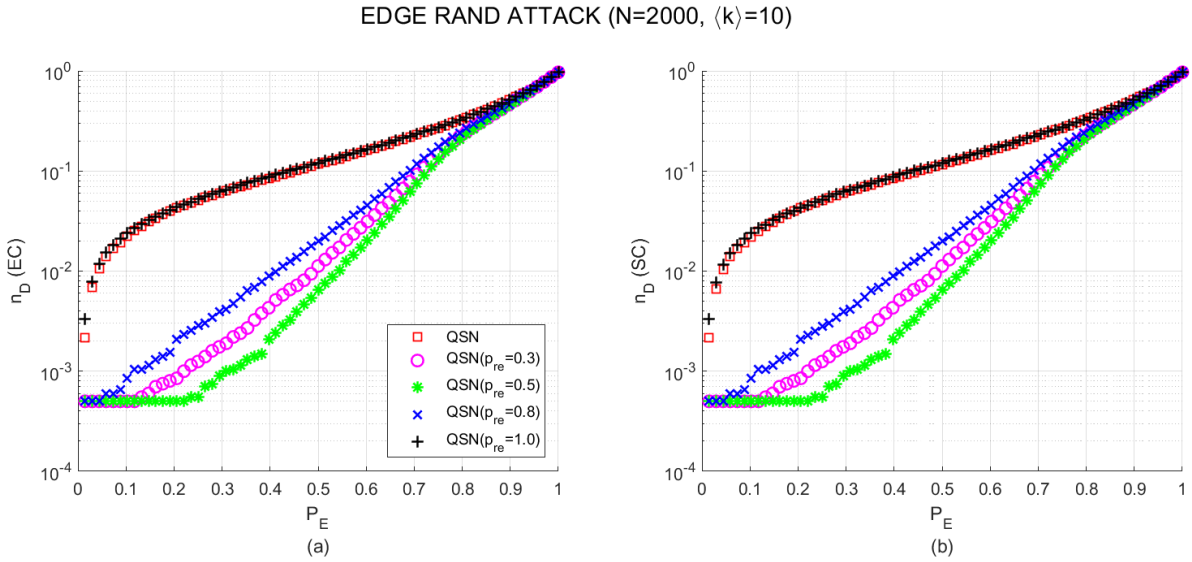


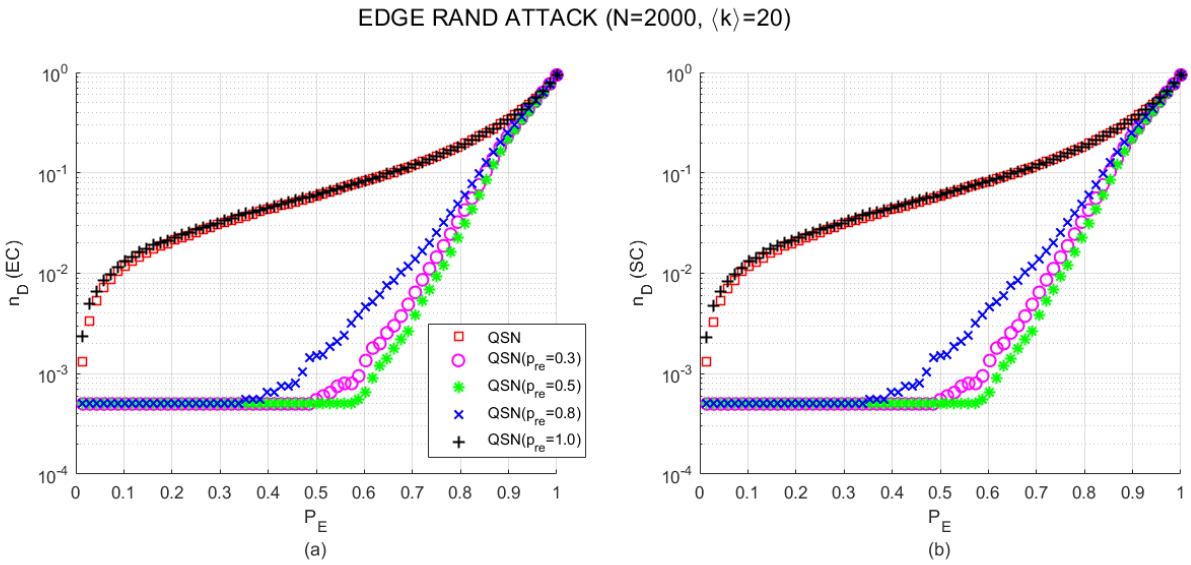

NODE TAR DEGREE (N=1000, ⟨k⟩=20)
QSN
QSN(p_re=0.3)
QSN(p_re=0.5)
QSN(p_re=0.8)
QSN(p_re=1.0)
n_D (EC)
n_D (SC)
P_N
(a)
(b)

4.3 Network Size N=2000

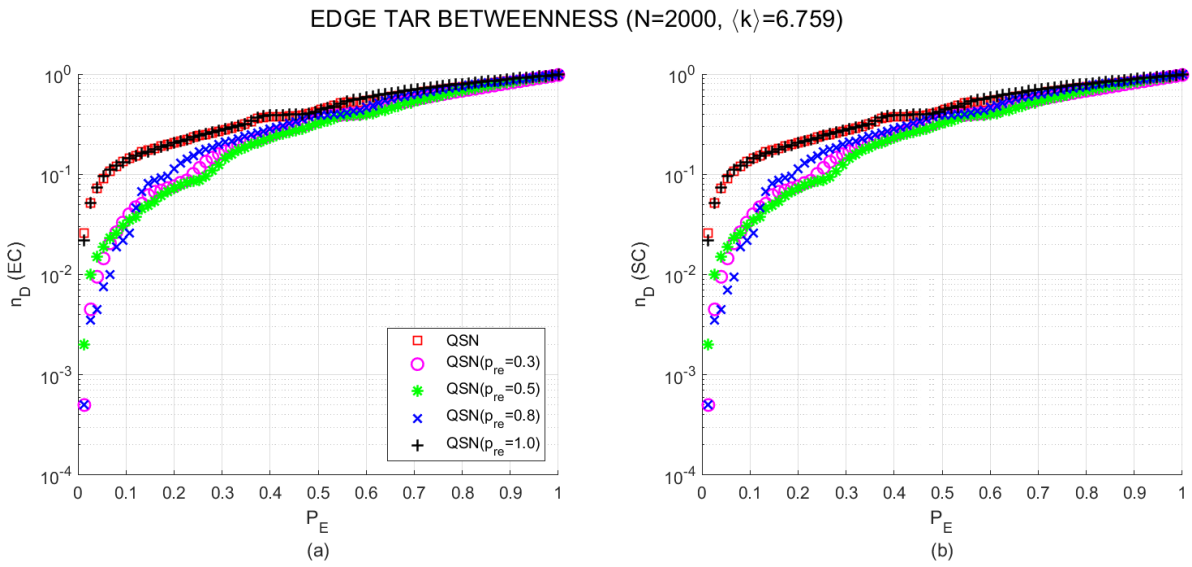
4.3.1 Edge Random Attack

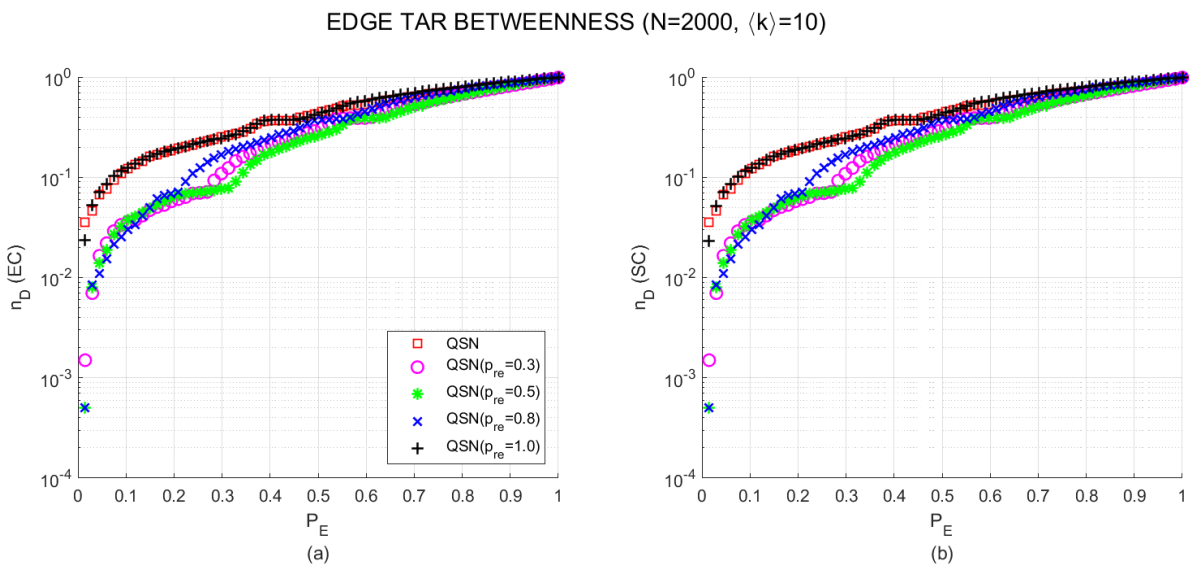




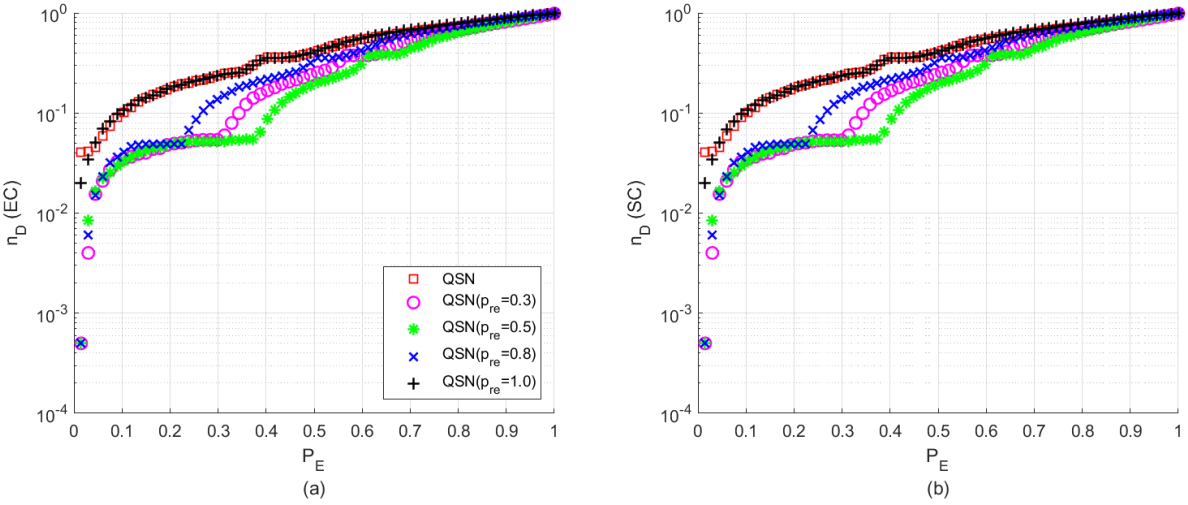

EDGE RAND ATTACK (N=2000, ⟨k⟩=20)
n_D (EC)
P_E
(a)
QSN
QSN(p_{re}=0.3)
QSN(p_{re}=0.5)
QSN(p_{re}=0.8)
QSN(p_{re}=1.0)
n_D (SC)
P_E
(b)

4.3.2 *Edge Intentional (Betweenness-based) Attack*

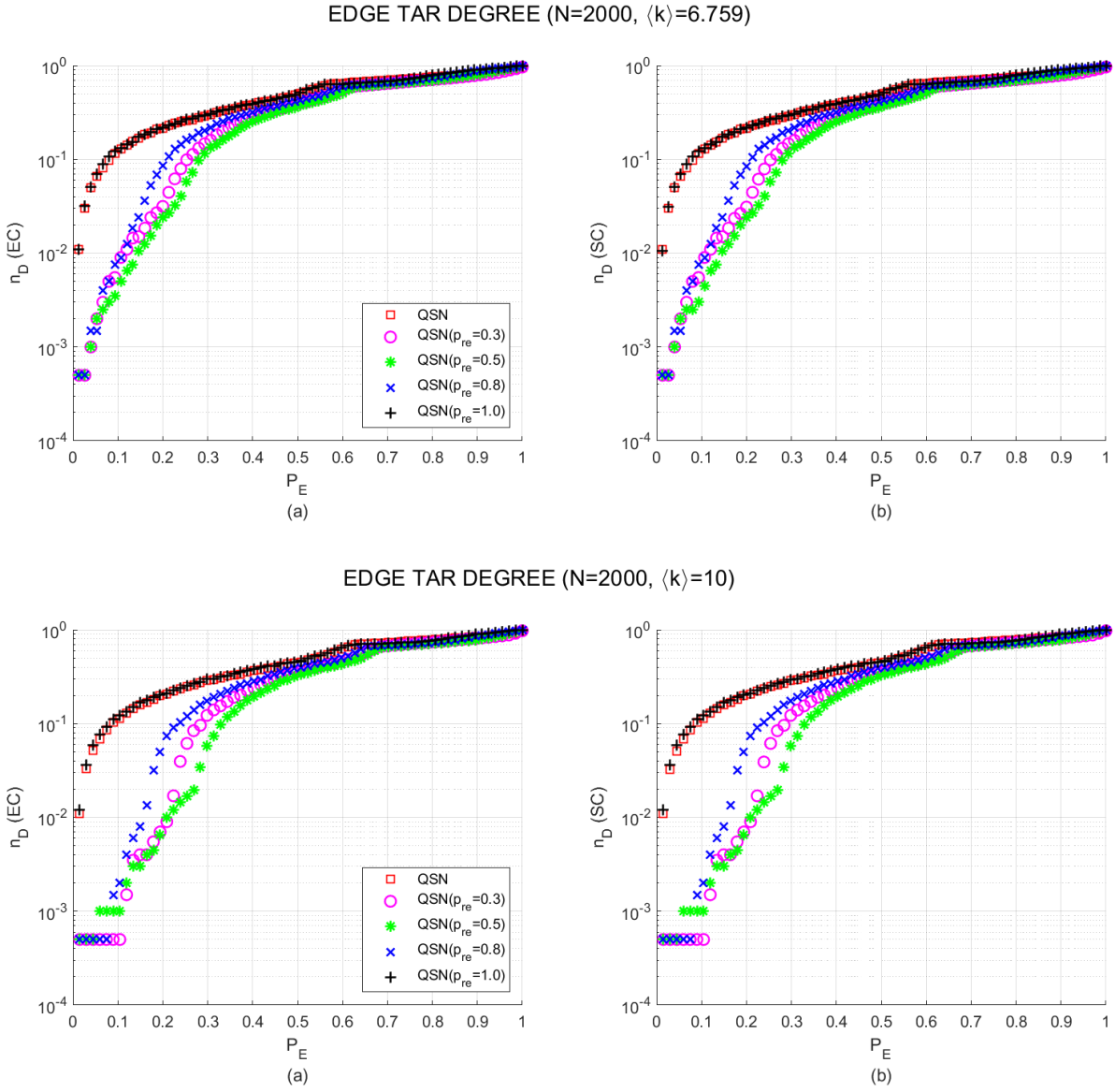


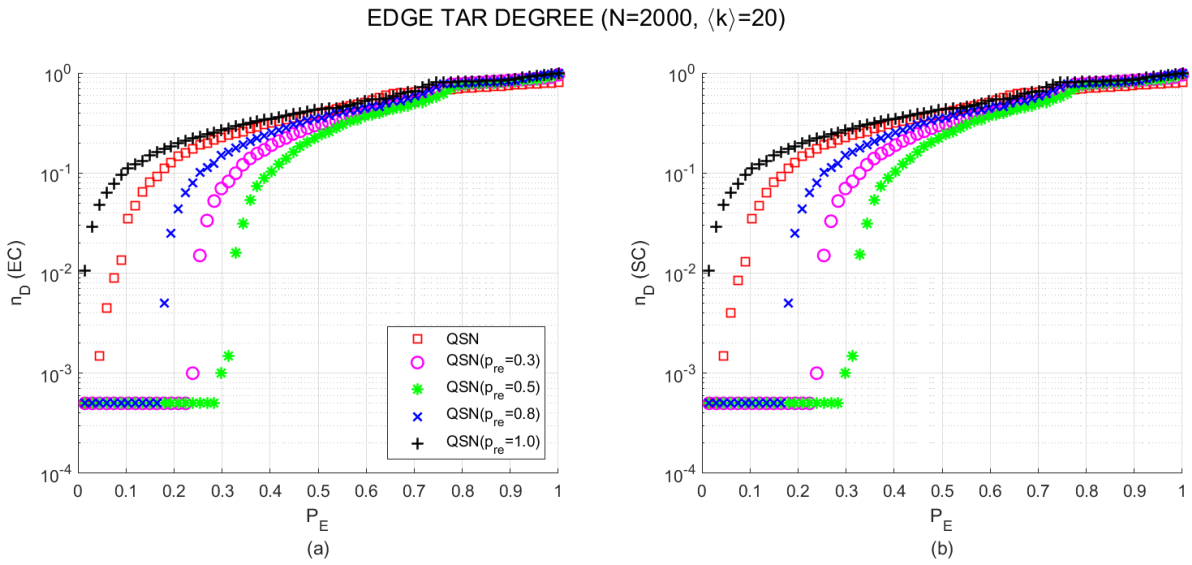


EDGE TAR BETWEENNESS (N=2000, ⟨k⟩=20)

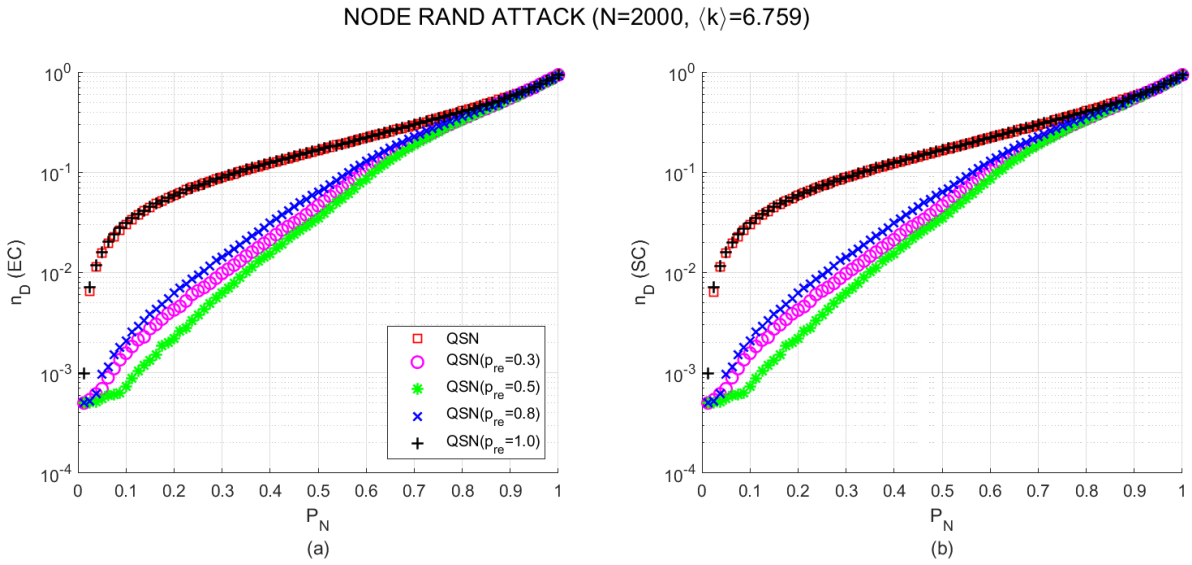


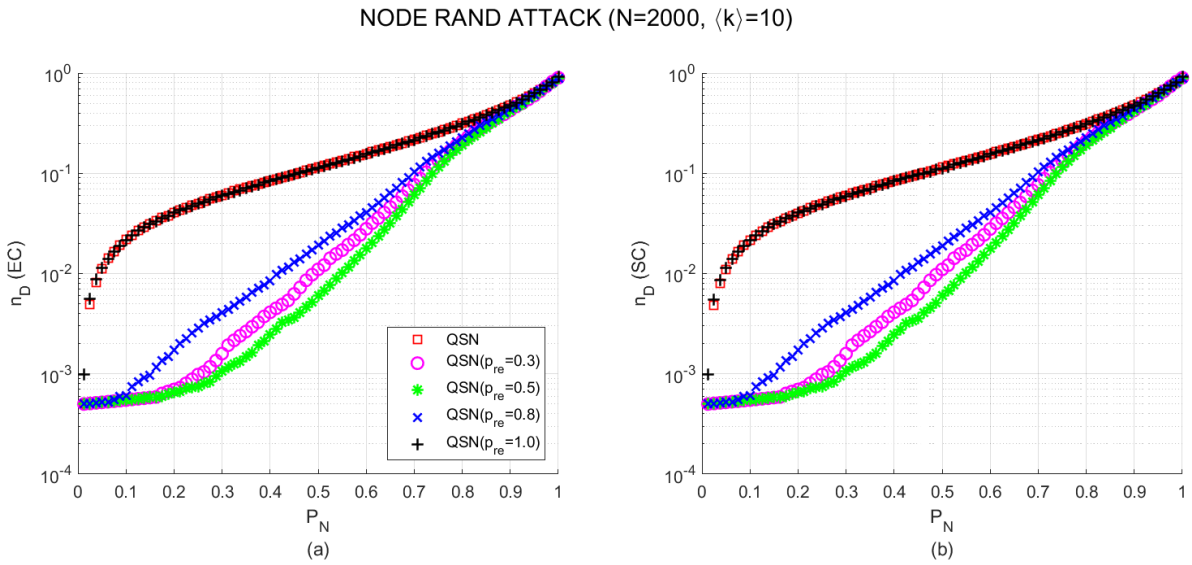
4.3.3 *Edge Intentional (Degree-based) Attack*

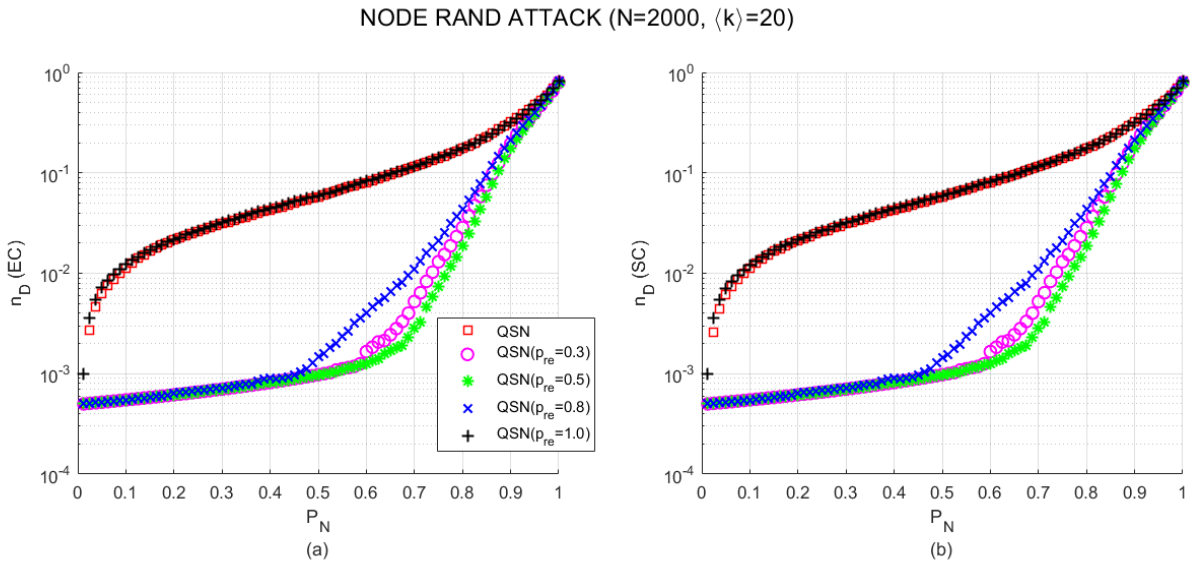



EDGE TAR DEGREE (N=2000, ⟨k⟩=20)
n_D (EC)
P_E
(a)
QSN
QSN(p_{re}=0.3)
QSN(p_{re}=0.5)
QSN(p_{re}=0.8)
QSN(p_{re}=1.0)
n_D (SC)
P_E
(b)

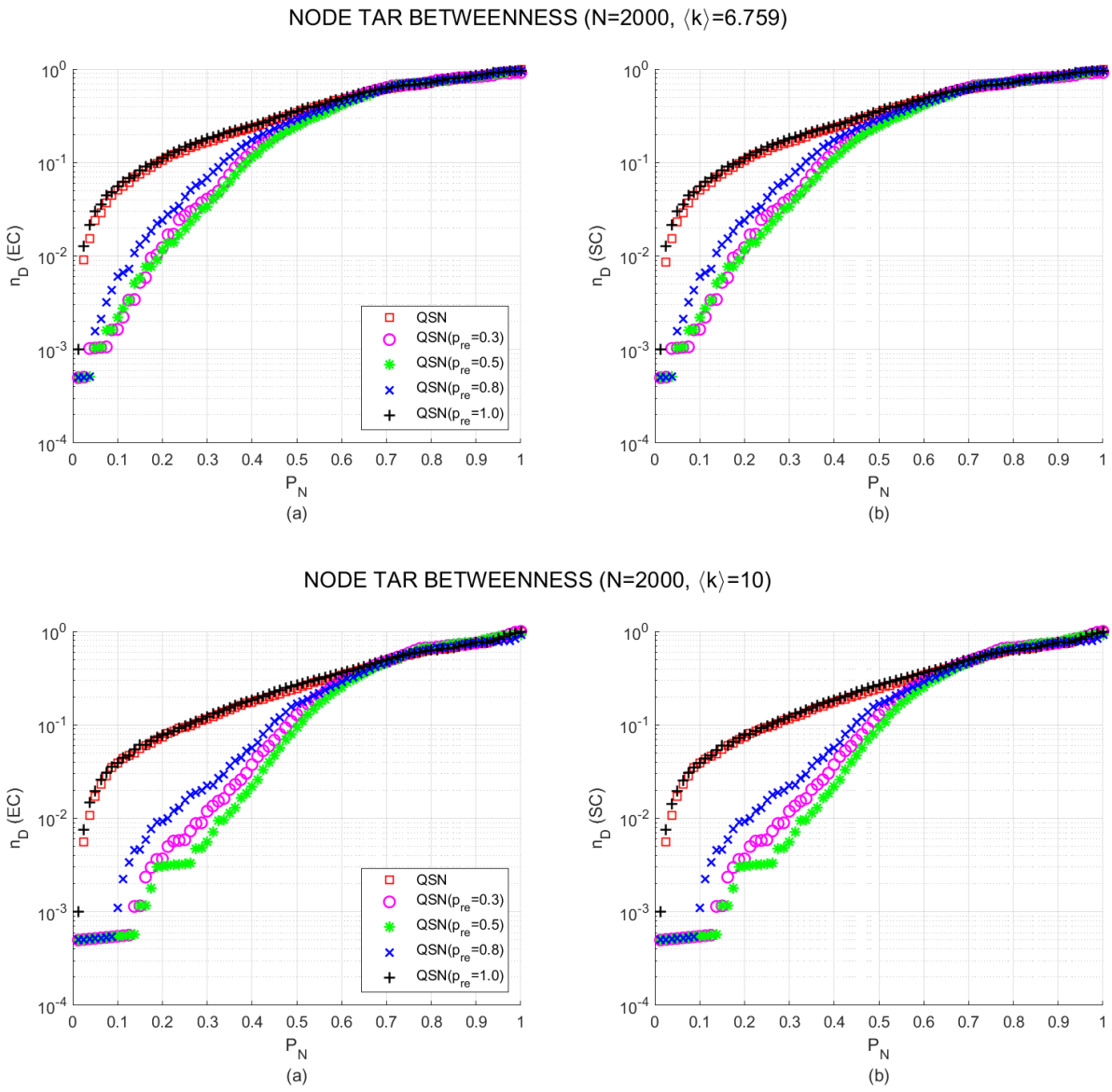
4.3.4 *Node Random Attack*

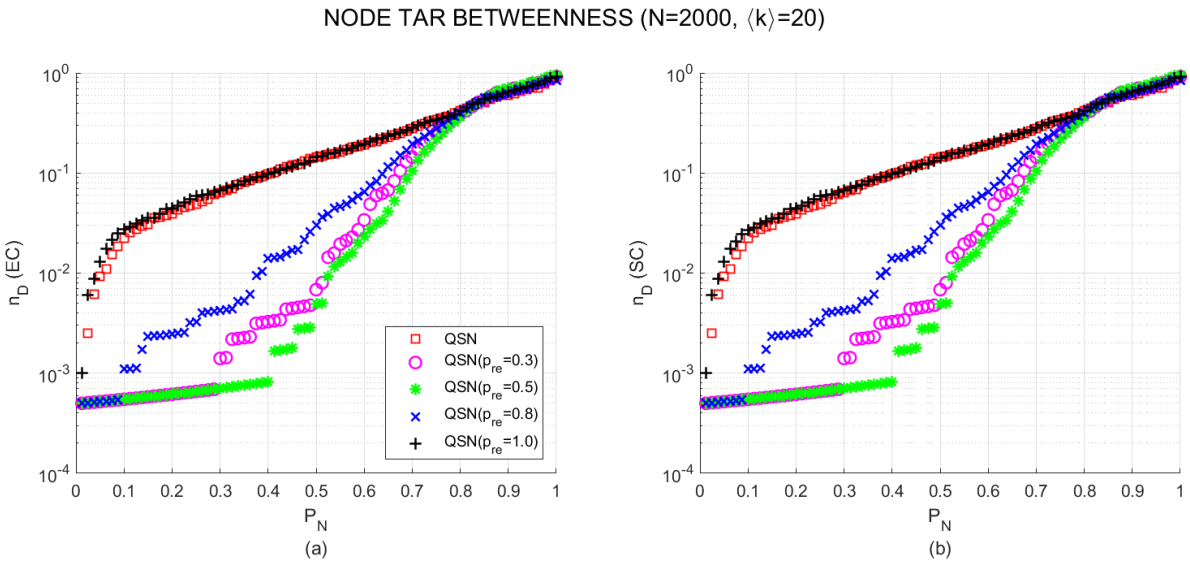




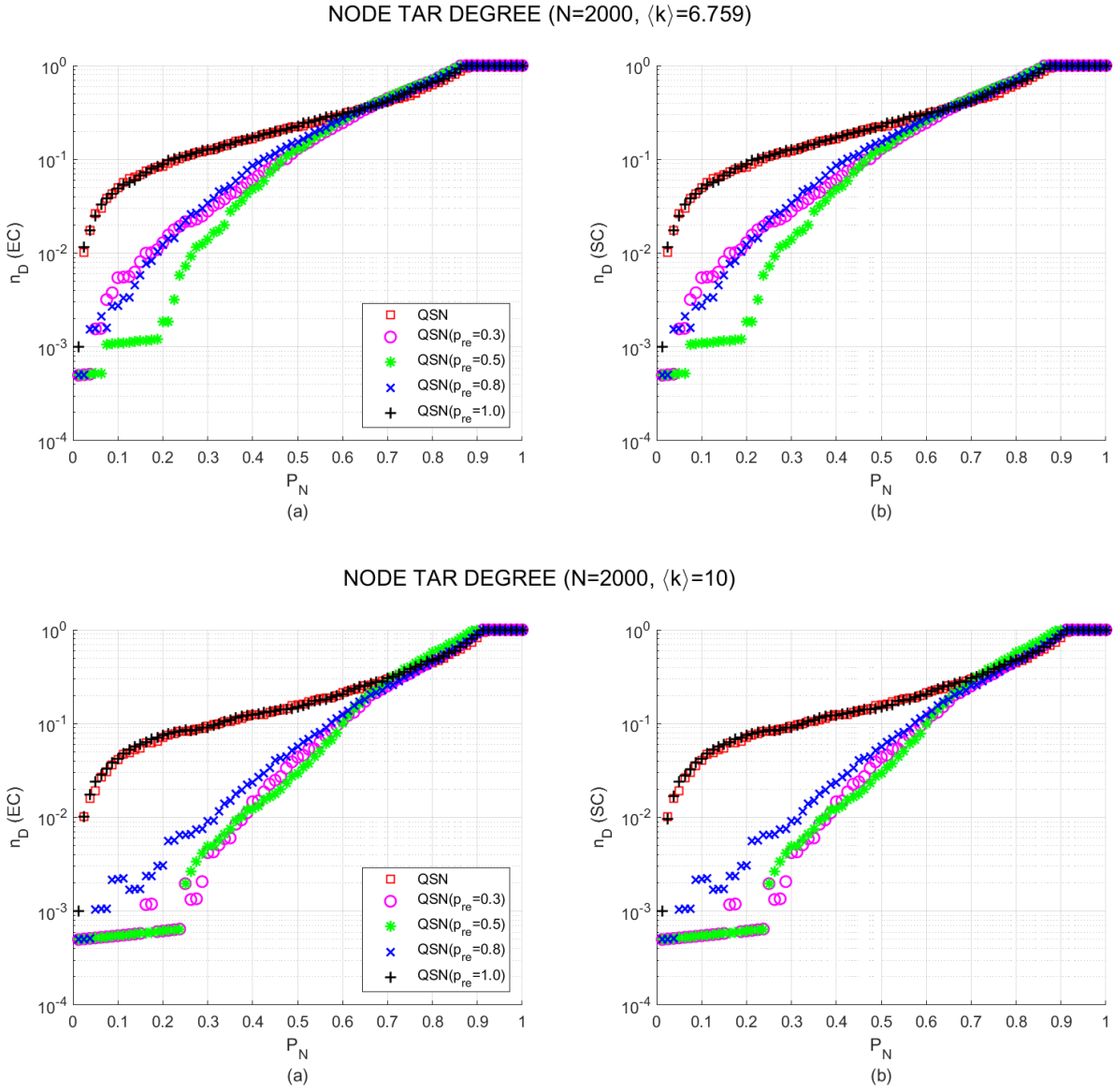

NODE RAND ATTACK (N=2000, ⟨k⟩=20)
n_D (EC)
P_N
(a)
QSN
QSN(p_{re}=0.3)
QSN(p_{re}=0.5)
QSN(p_{re}=0.8)
QSN(p_{re}=1.0)
n_D (SC)
P_N
(b)

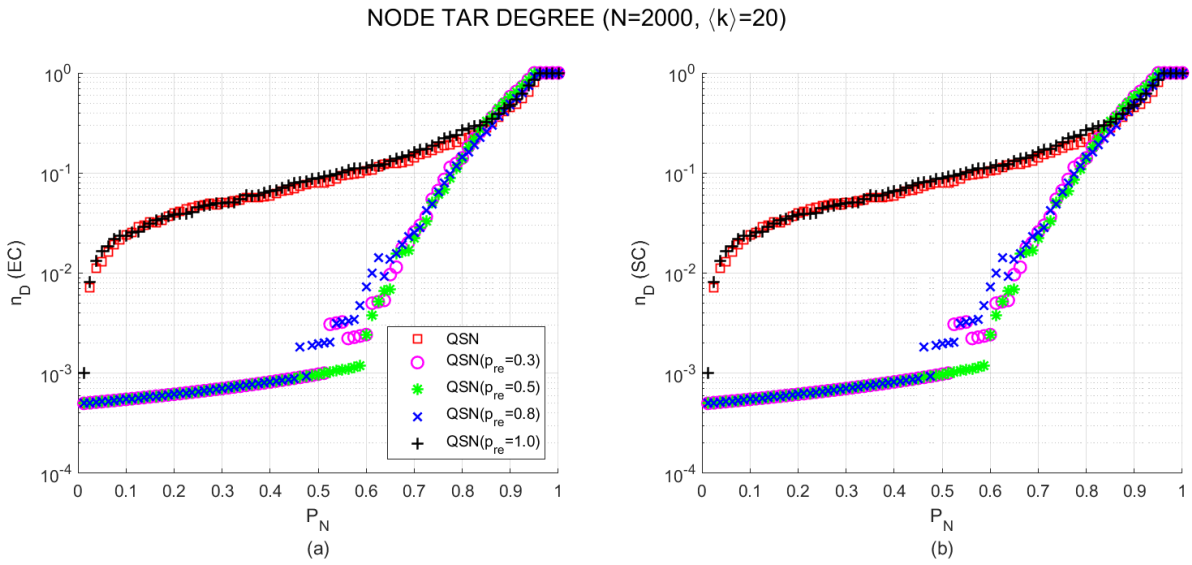
4.3.5 *Node Intentional (Betweenness-based) Attack*




NODE TAR BETWEENNESS (N=2000, ⟨k⟩=20)
n_D (EC)
P_N
(a)
QSN
QSN(p_{re}=0.3)
QSN(p_{re}=0.5)
QSN(p_{re}=0.8)
QSN(p_{re}=1.0)
n_D (SC)
P_N
(b)

4.3.6 *Node Intentional (Degree-based) Attack*

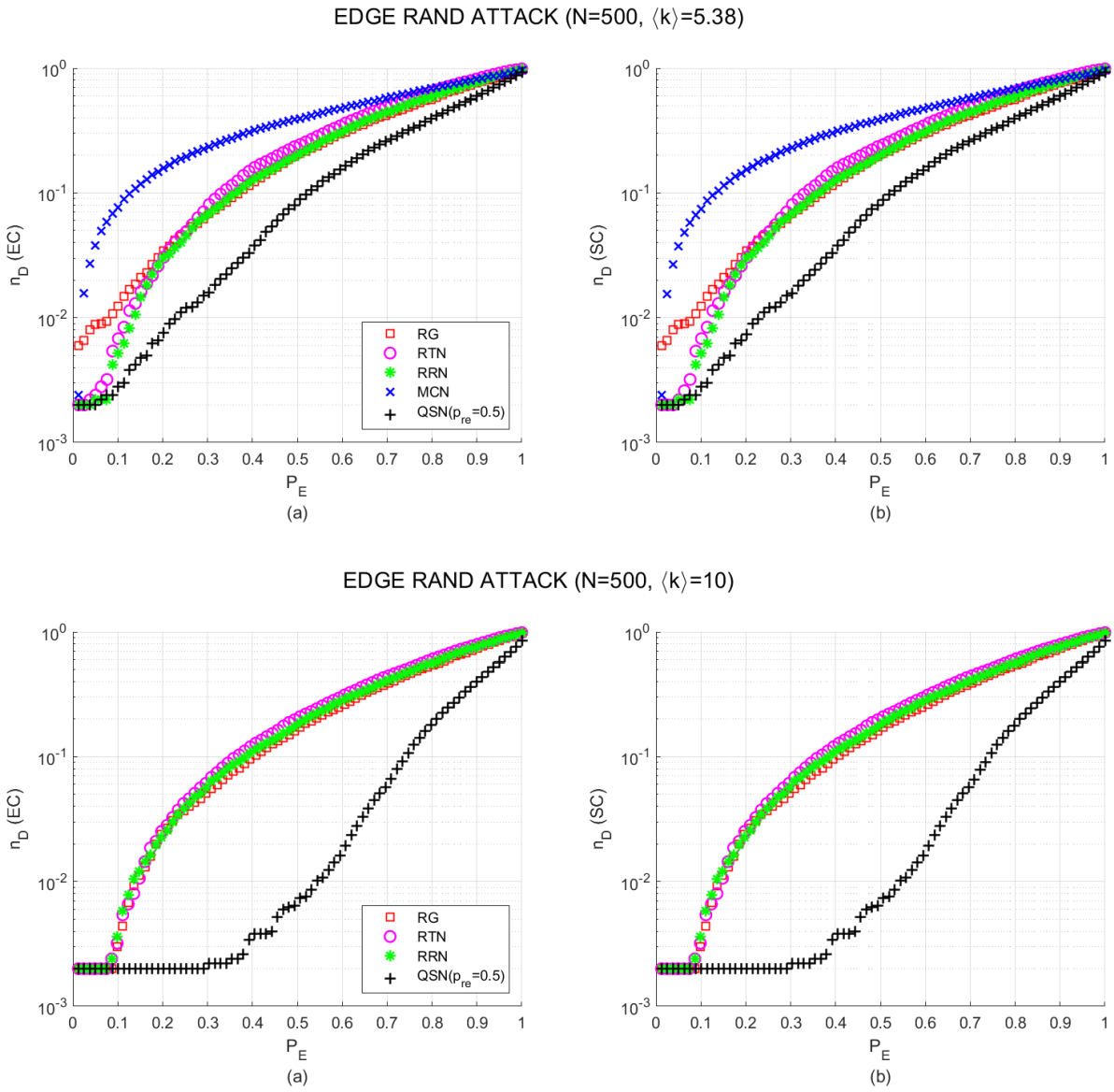



NODE TAR DEGREE (N=2000, ⟨k⟩=20)
n_D (EC)
P_N
(a)
QSN
QSN(p_{re}=0.3)
QSN(p_{re}=0.5)
QSN(p_{re}=0.8)
QSN(p_{re}=1.0)
n_D (SC)
P_N
(b)

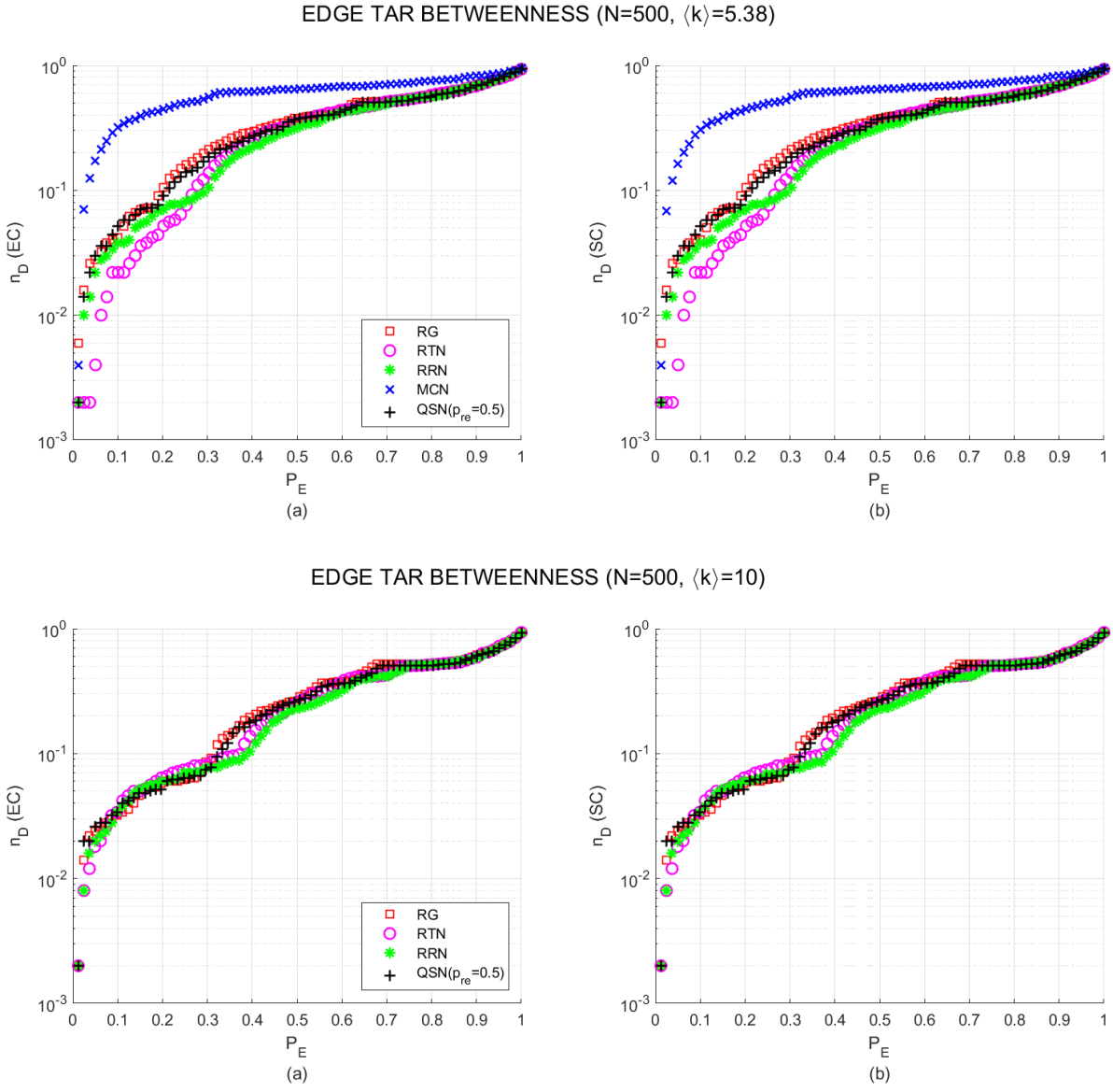
5 QSN with $p_{re} = 0.5$ vs. Other Network Topologies Comparison Curves

5.1 Network Size N=500

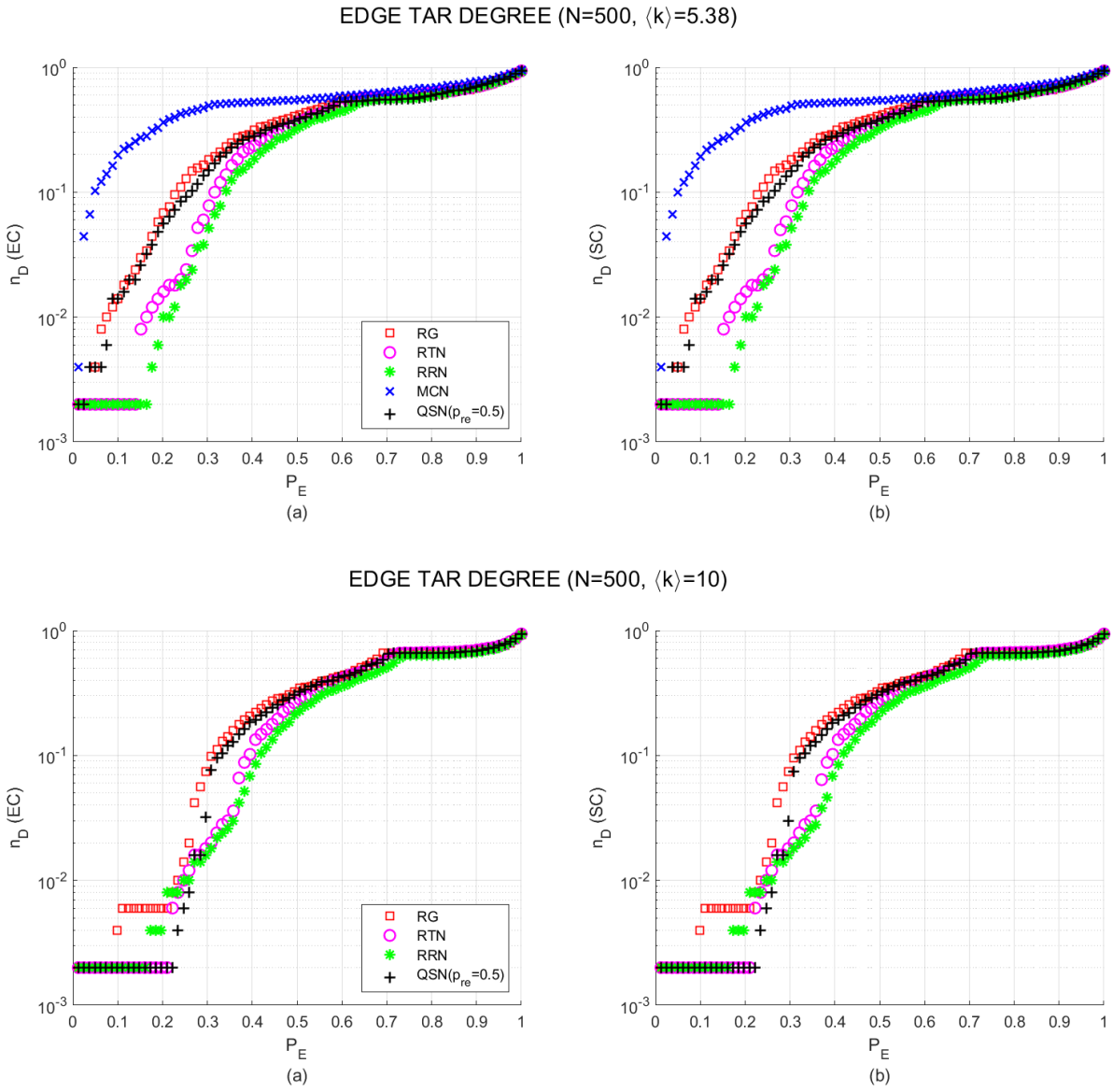
5.1.1 Edge Random Attack



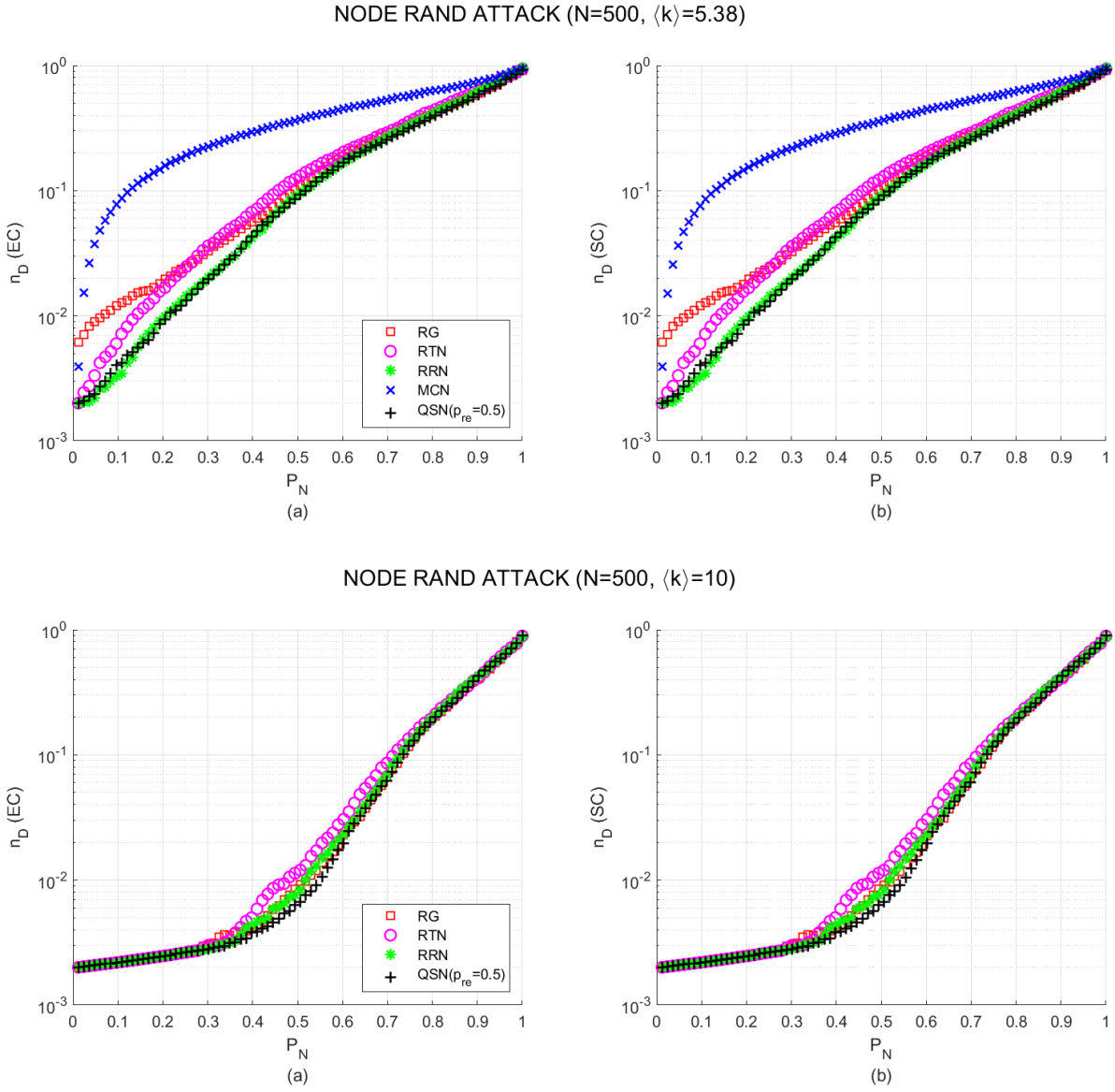
5.1.2 *Edge Intentional (Betweenness-based) Attack*



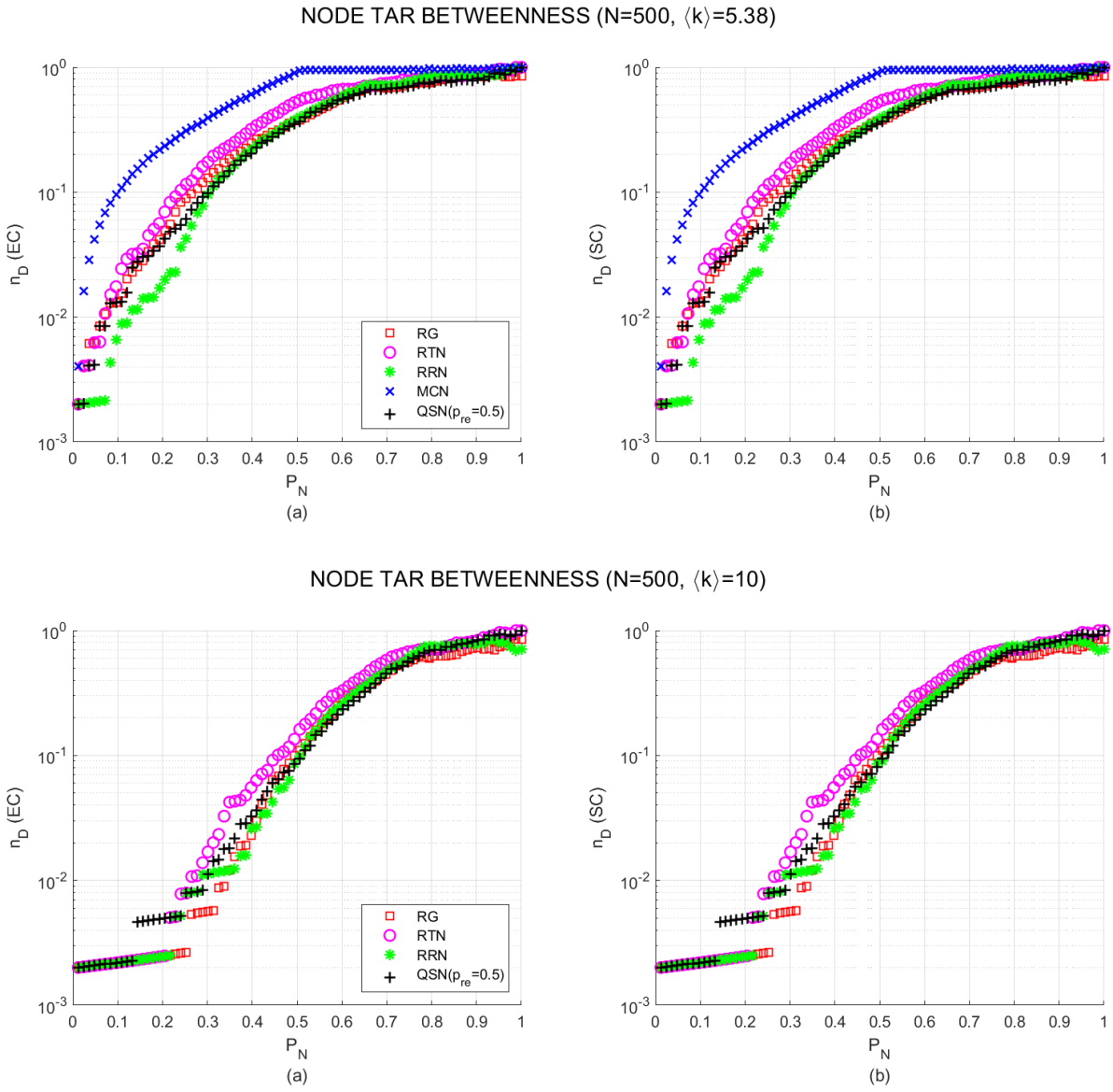
5.1.3 Edge Intentional (Degree-based) Attack



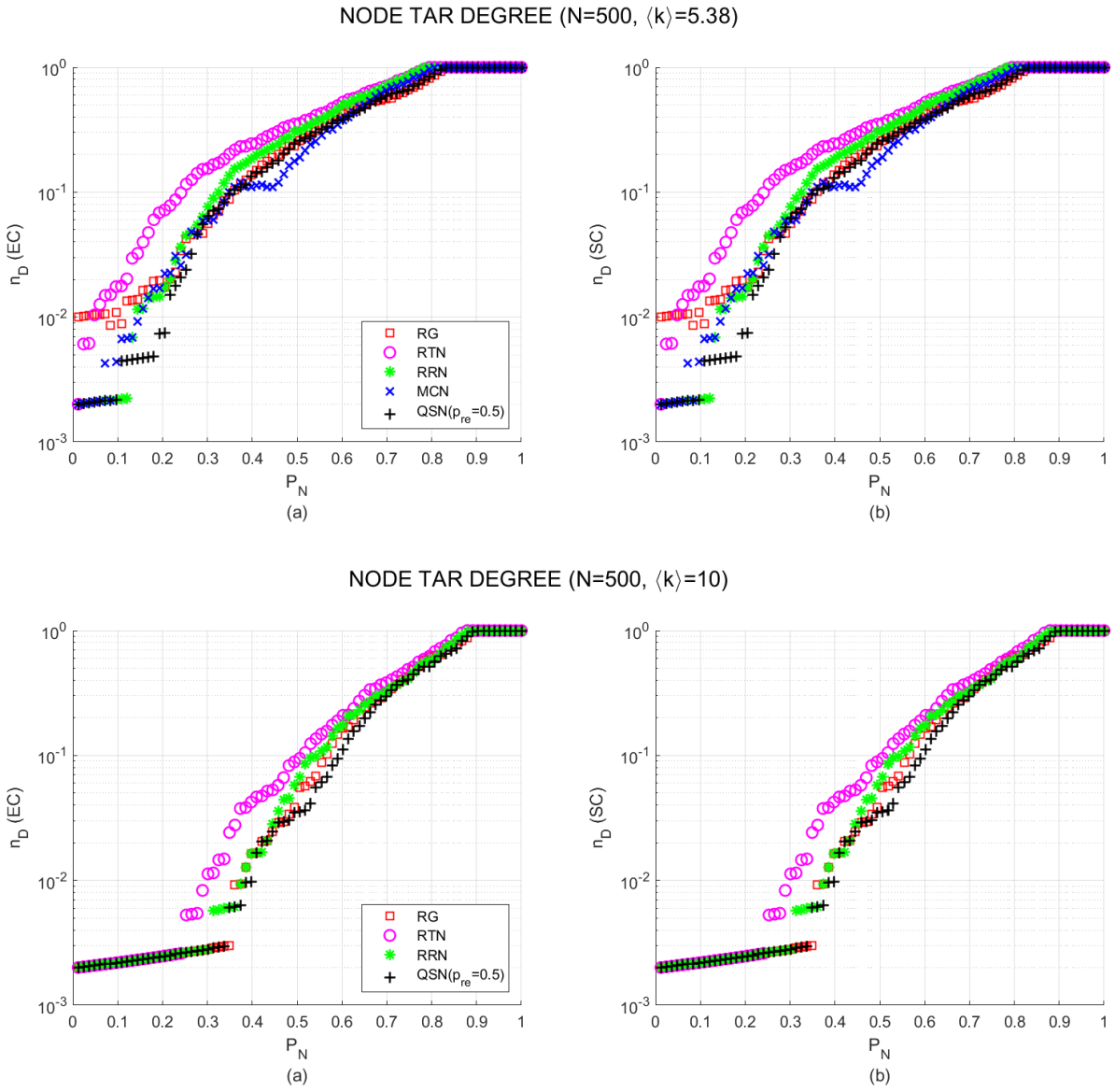
5.1.4 *Node Random Attack*



5.1.5 *Node Intentional (Betweenness-based) Attack*

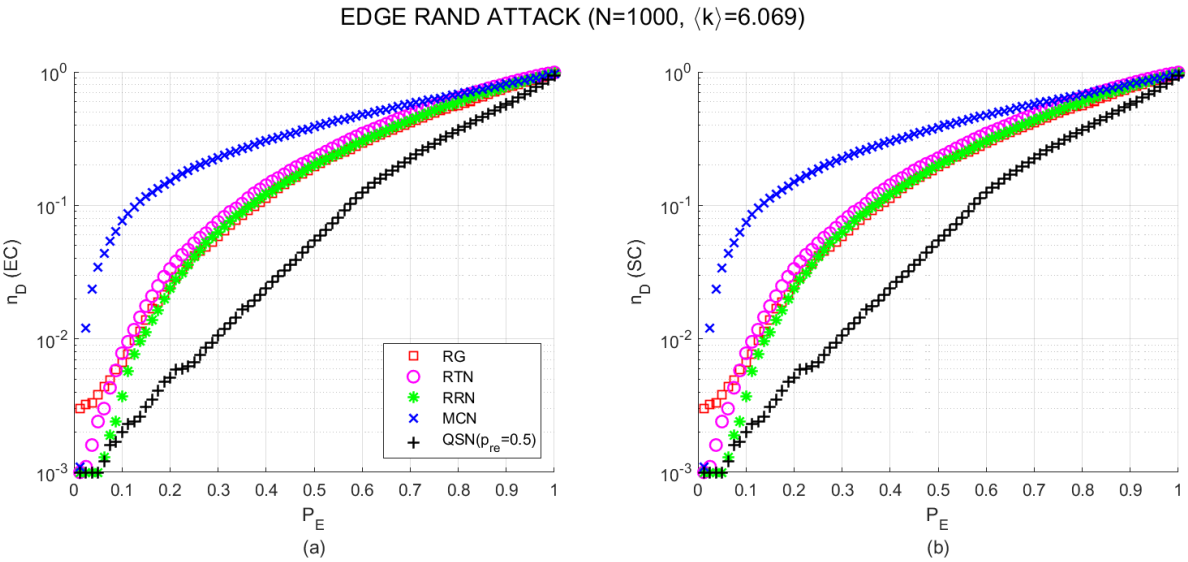


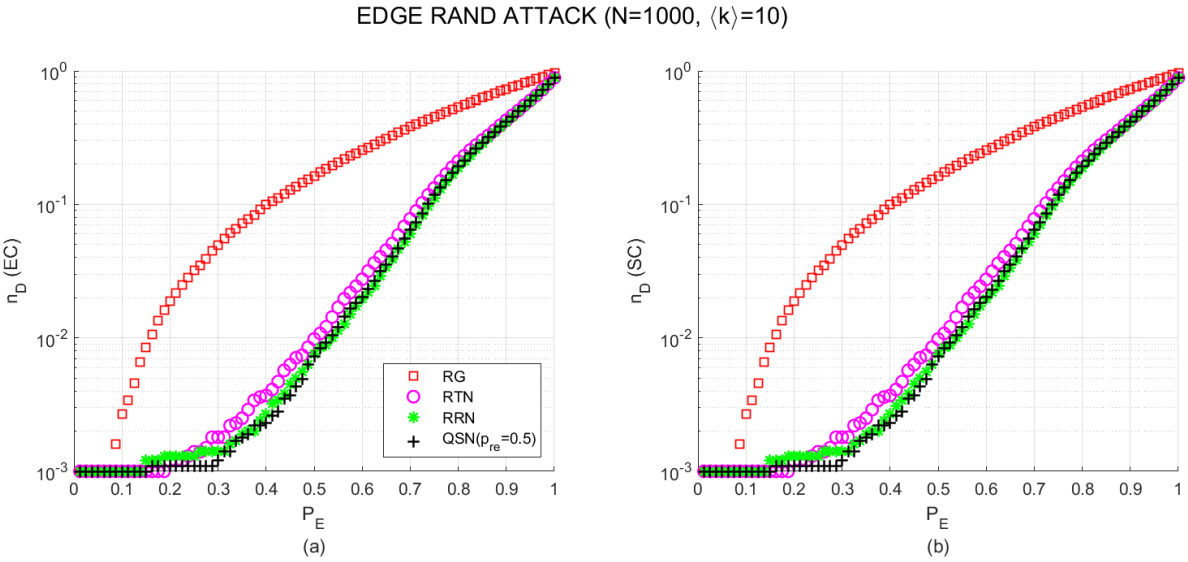
5.1.6 *Node Intentional (Degree-based) Attack*

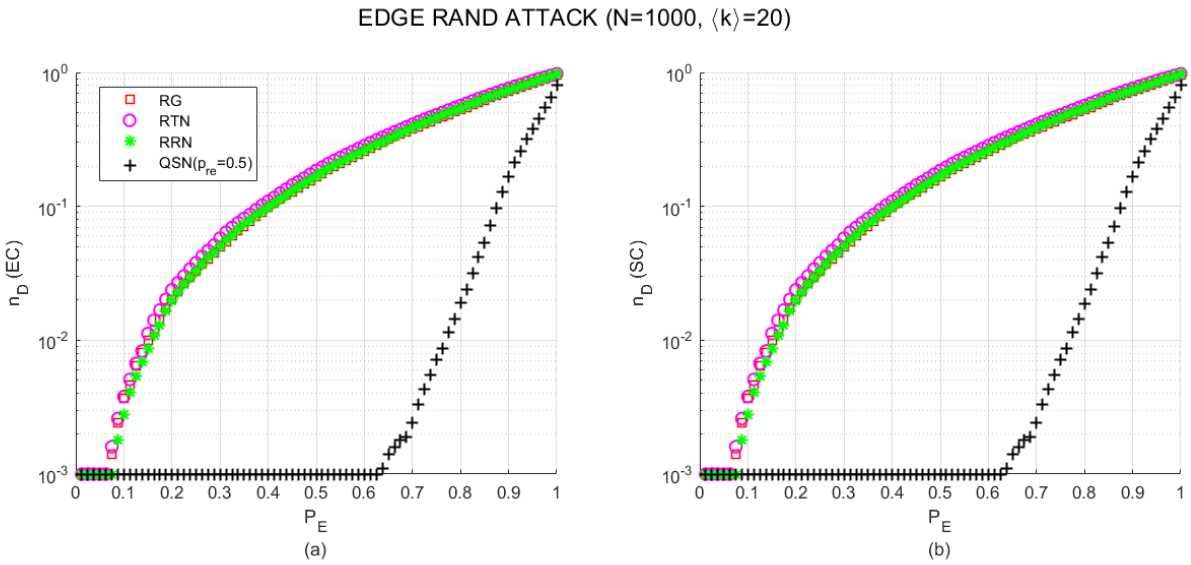


5.2 Network Size N=1000

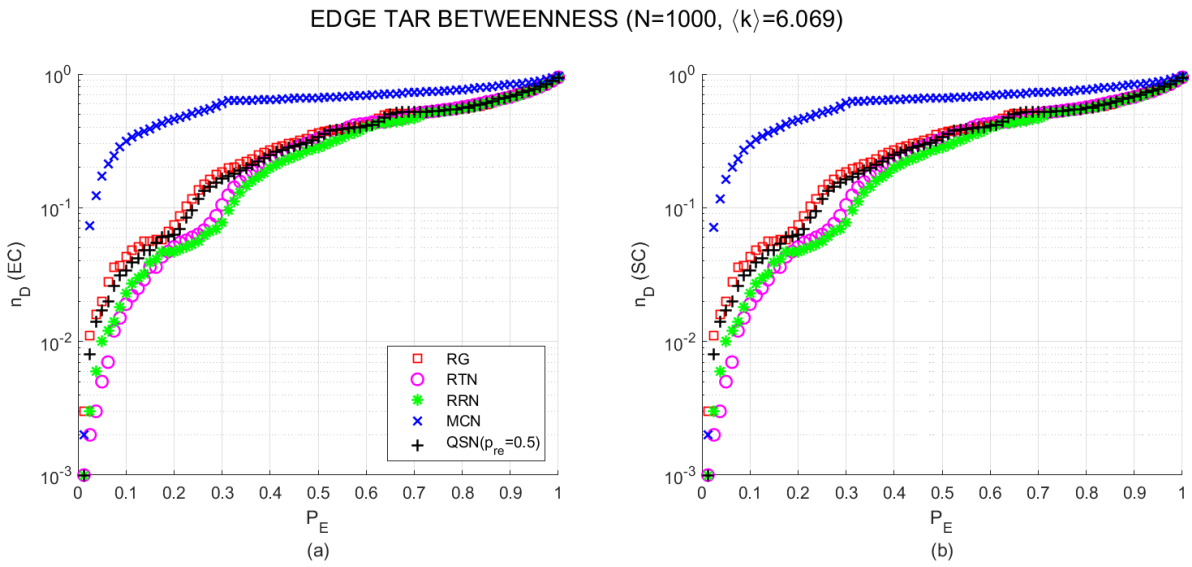
5.2.1 Edge Random Attack

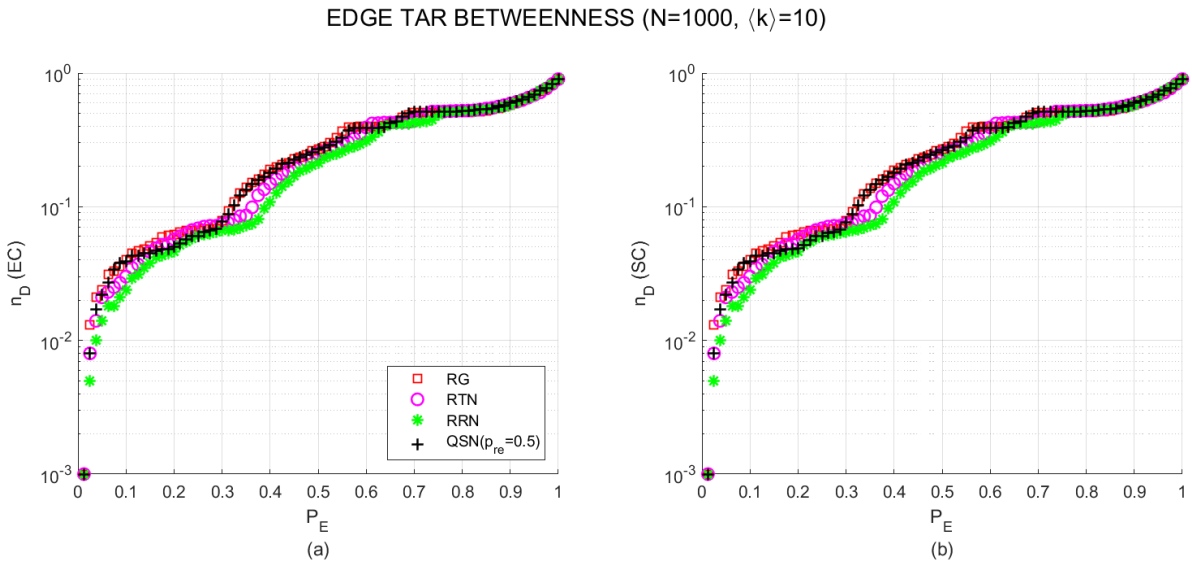


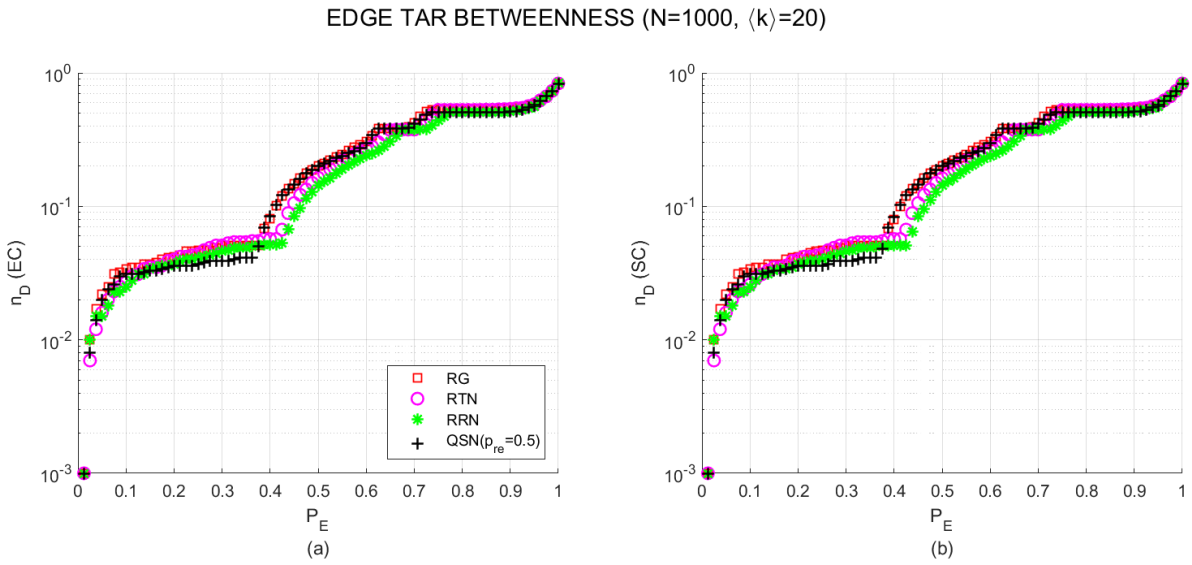



EDGE RAND ATTACK (N=1000, ⟨k⟩=20)
RG
RTN
RRN
QSN(p_re=0.5)
n_D (EC)
n_D (SC)
P_E
(a)
(b)

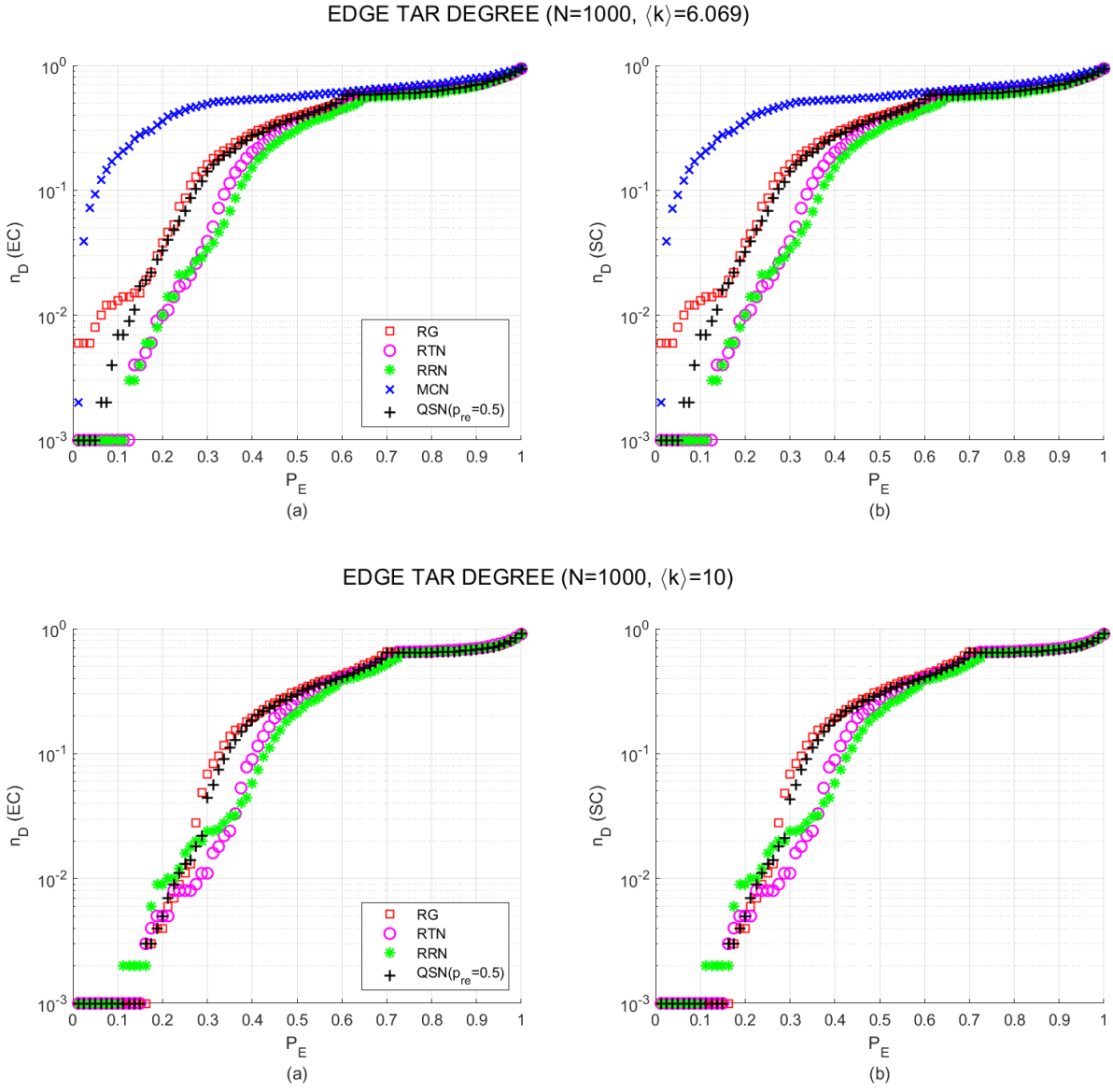
5.2.2 *Edge Intentional (Betweenness-based) Attack*

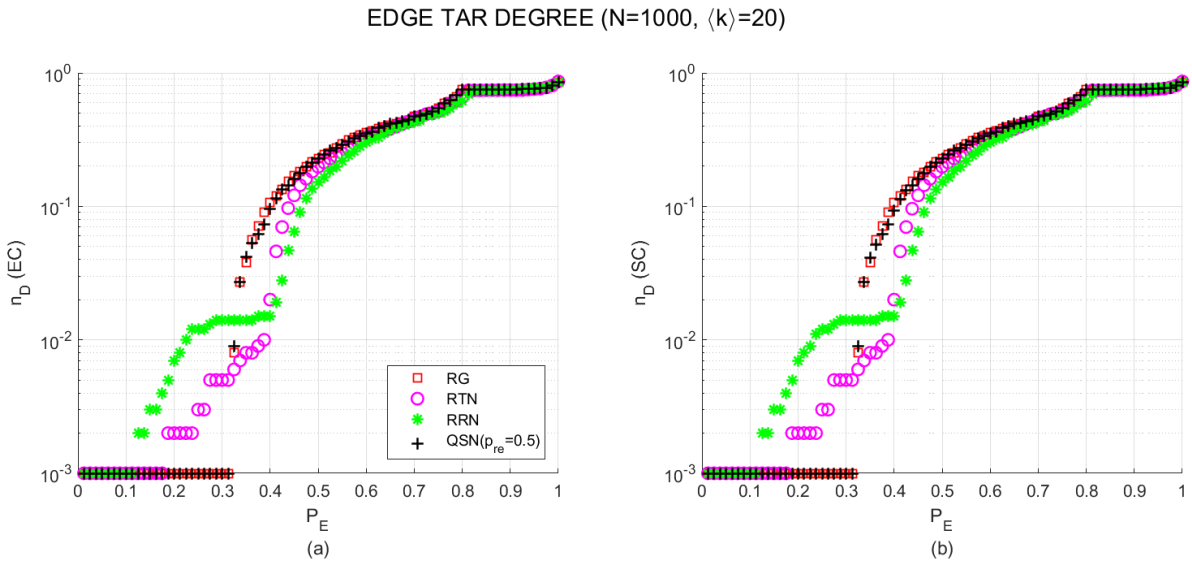




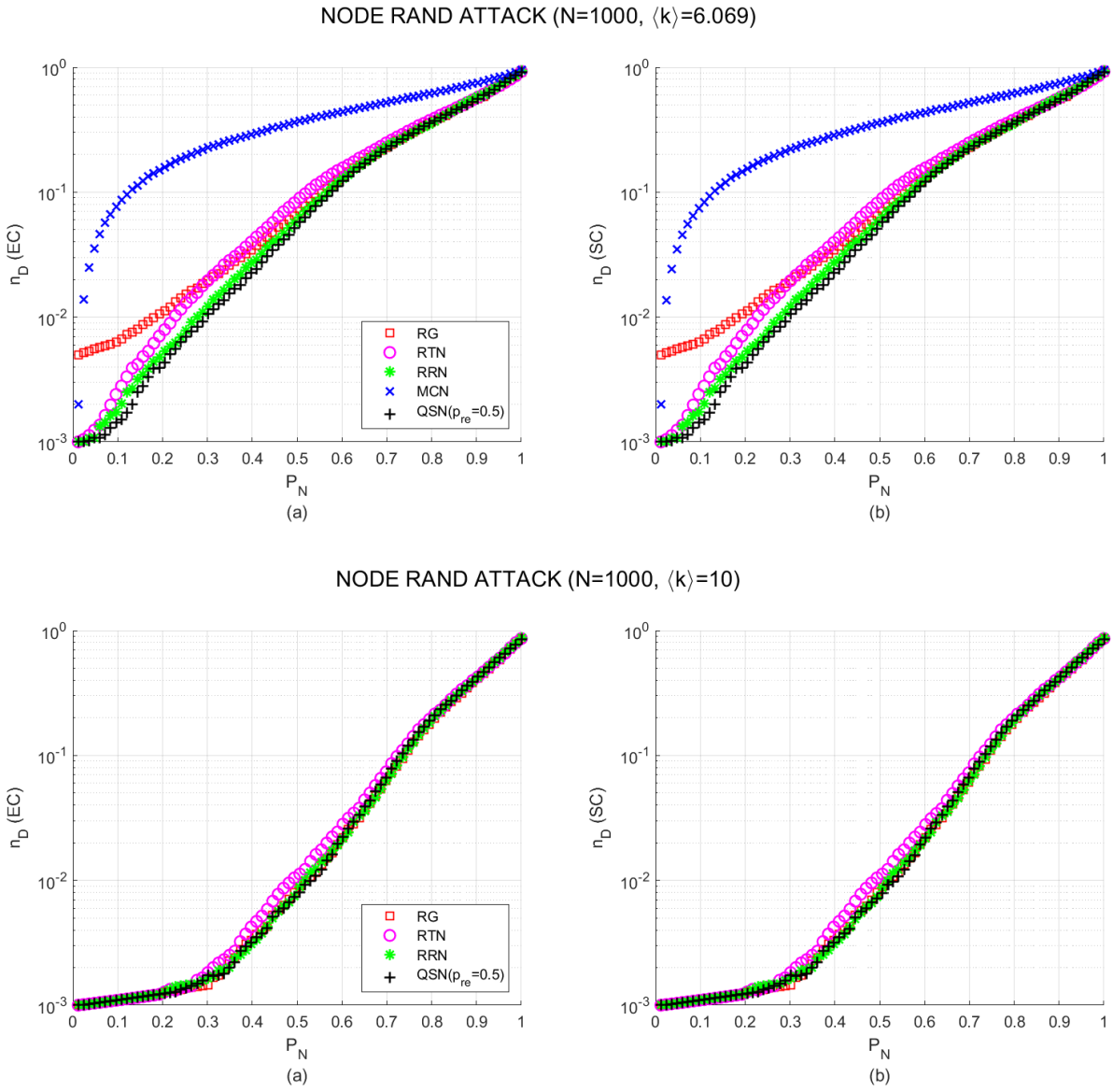

EDGE TAR BETWEENNESS (N=1000, ⟨k⟩=20)
n_D (EC)
P_E
(a)
RG
RTN
RRN
QSN(p_{re}=0.5)
n_D (SC)
P_E
(b)

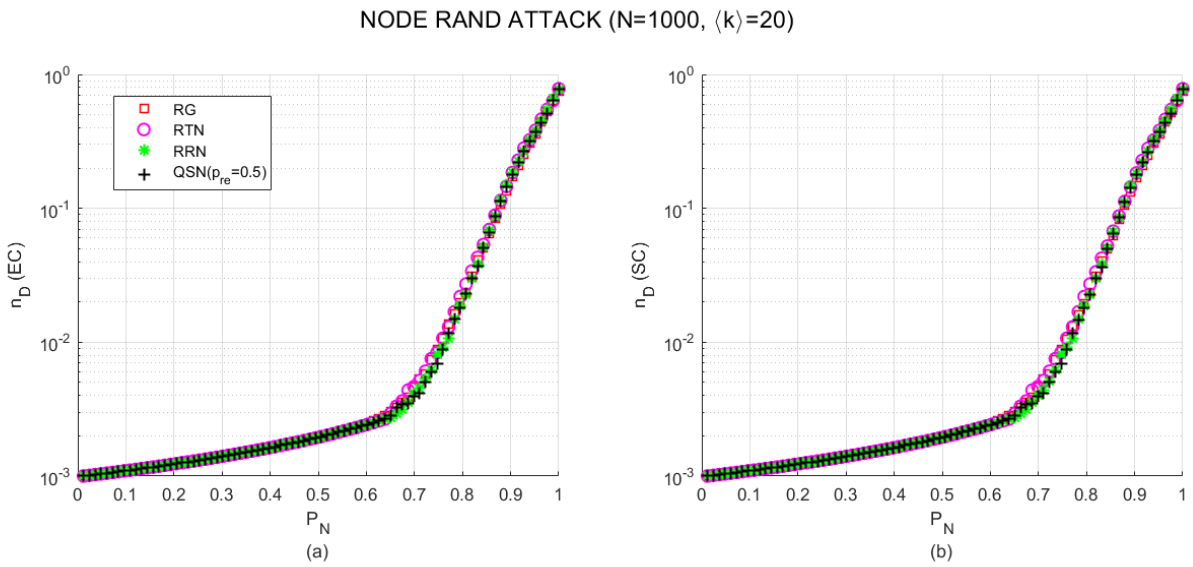
5.2.3 *Edge Intentional (Degree-based) Attack*



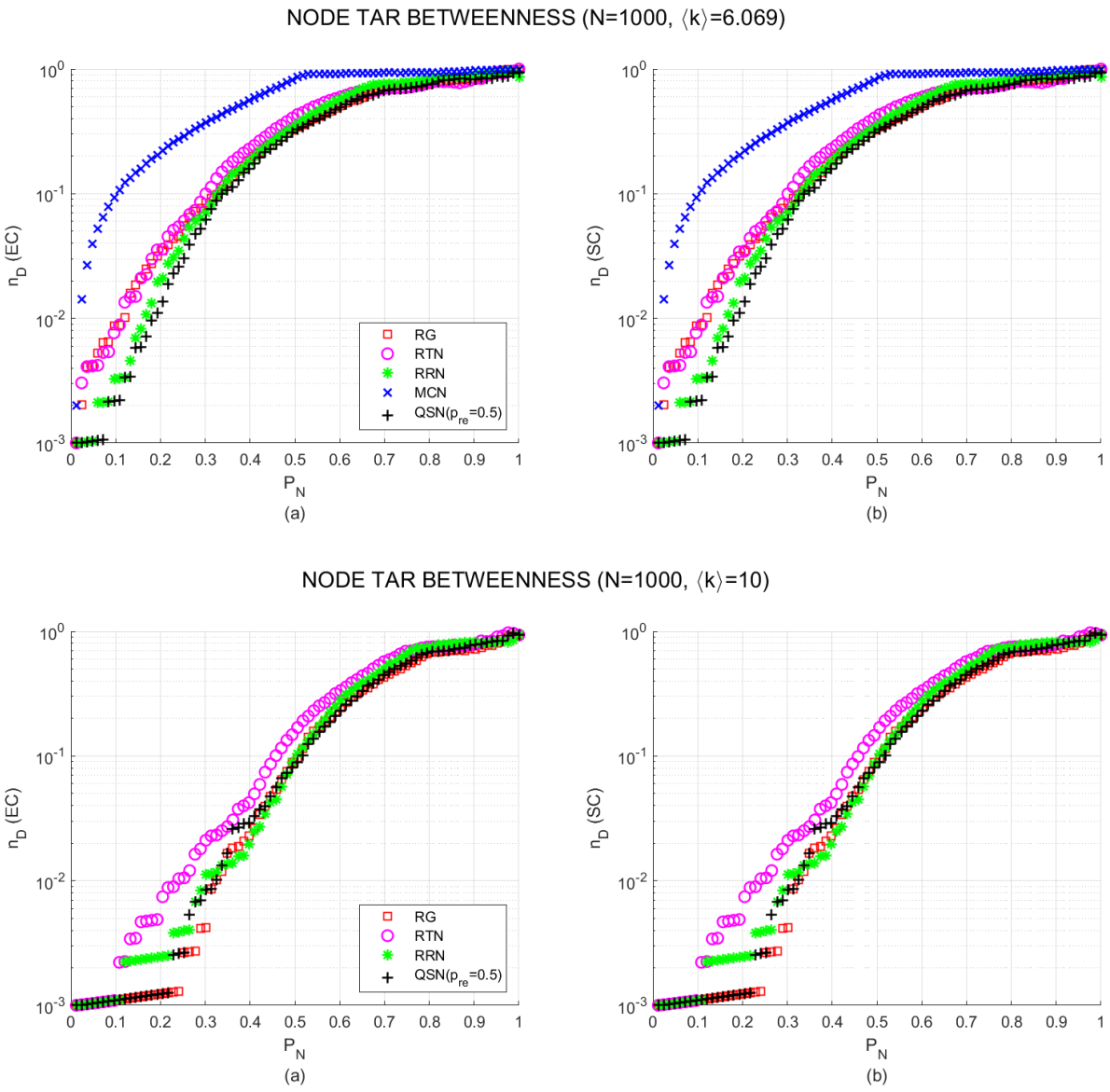

EDGE TAR DEGREE (N=1000, ⟨k⟩=20)
n_D (EC)
P_E
(a)
RG
RTN
RRN
QSN(p_{re}=0.5)
n_D (SC)
P_E
(b)

5.2.4 *Node Random Attack*

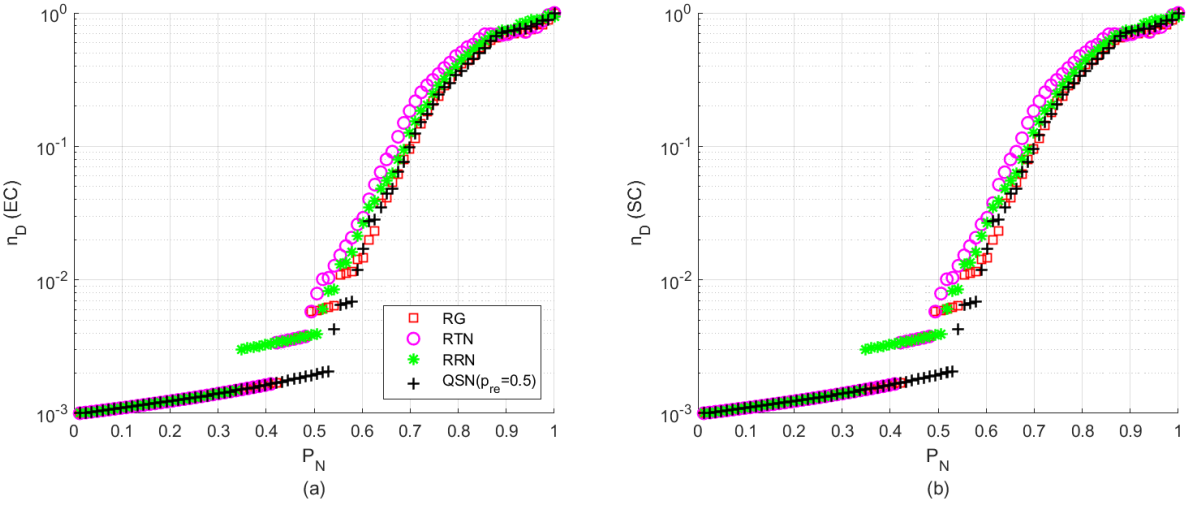



NODE RAND ATTACK (N=1000, ⟨k⟩=20)
RG
RTN
RRN
QSN(p_re=0.5)
n_D (EC)
P_N
(a)
n_D (SC)
P_N
(b)

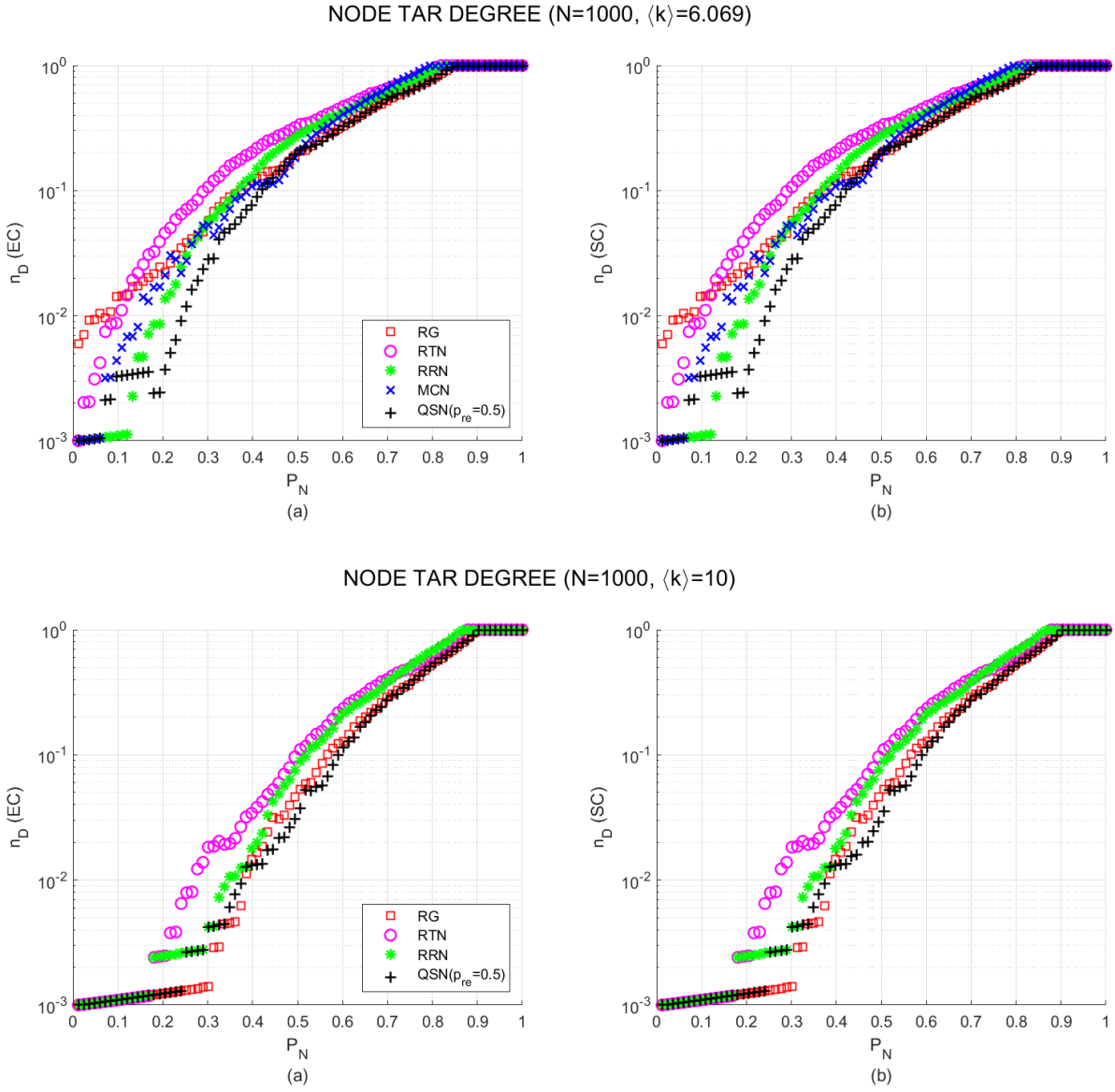
5.2.5 *Node Intentional (Betweenness-based) Attack*

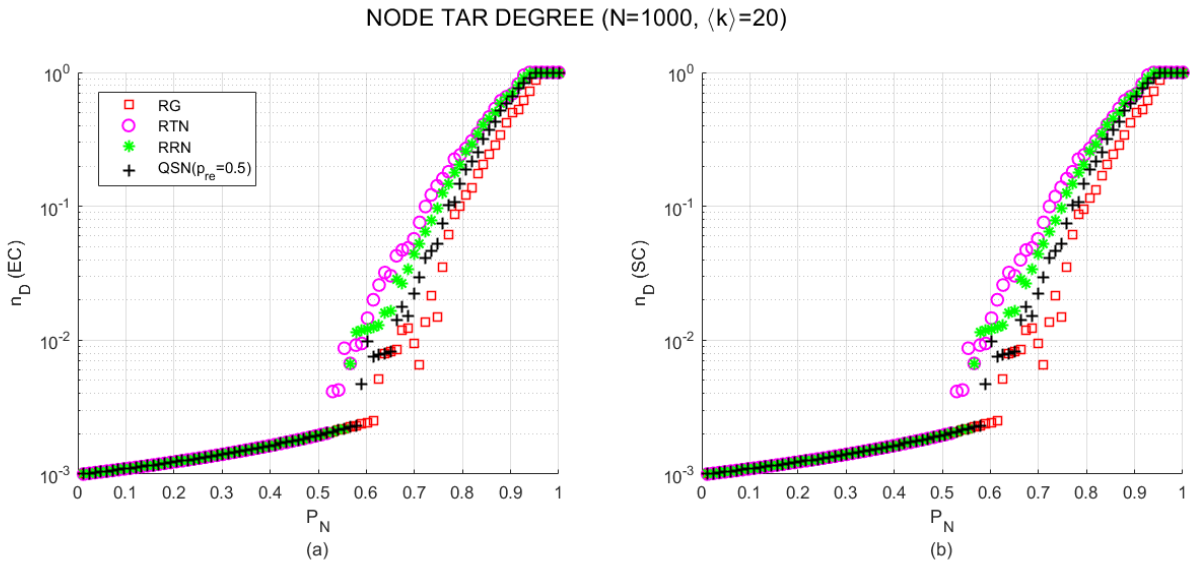


NODE TAR BETWEENNESS (N=1000, ⟨k⟩=20)



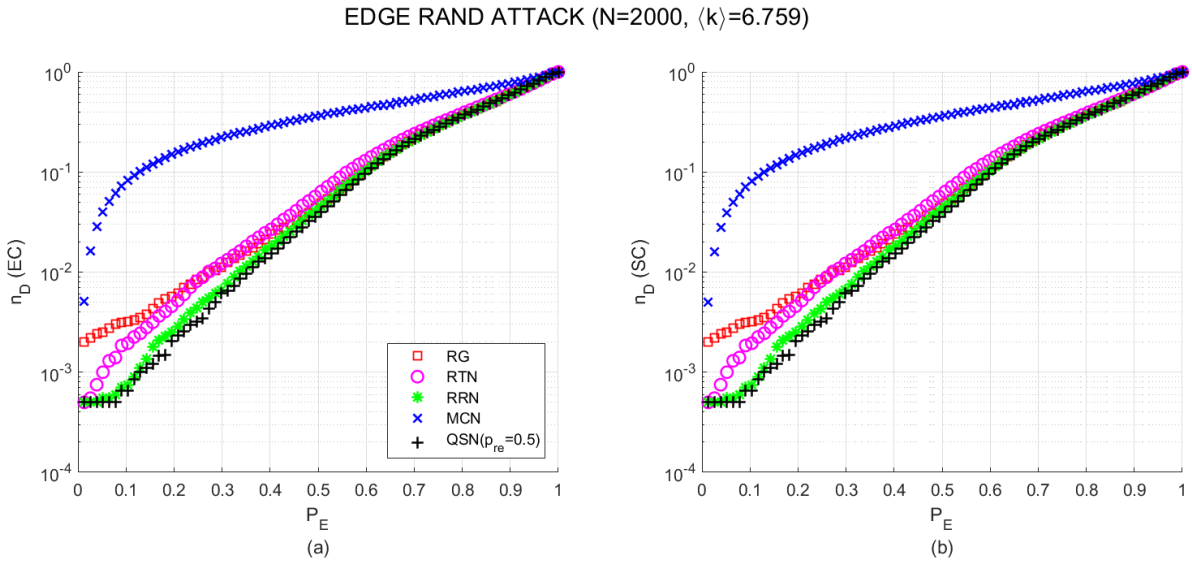
5.2.6 *Node Intentional (Degree-based) Attack*

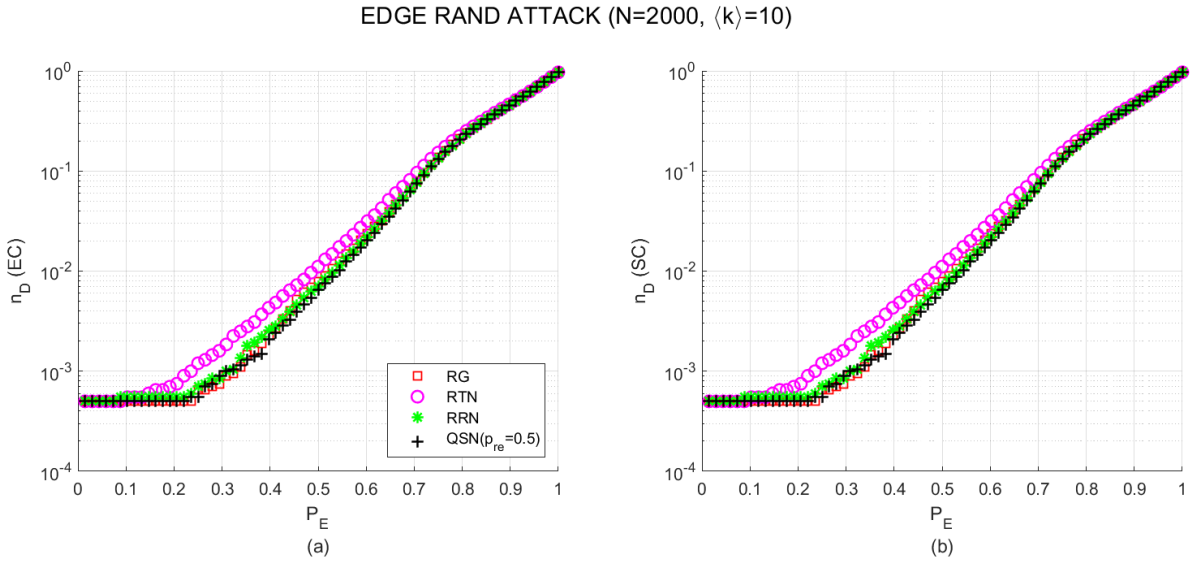


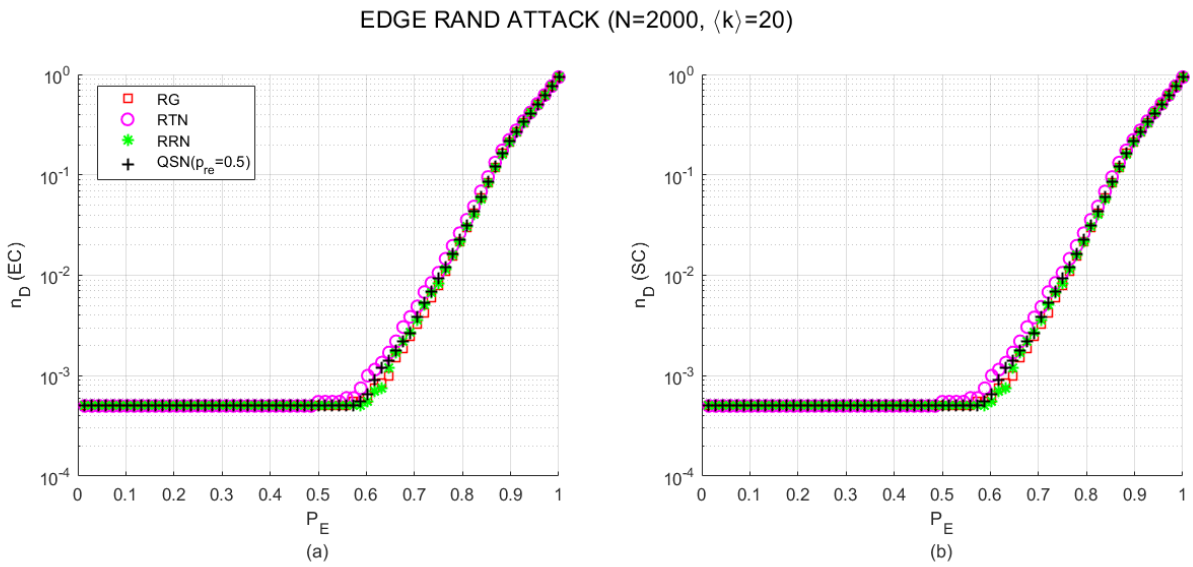

NODE TAR DEGREE (N=1000, ⟨k⟩=20)
RG
RTN
RRN
QSN(p_re=0.5)
n_D (EC)
P_N
(a)
n_D (SC)
P_N
(b)

5.3 Network Size N=2000

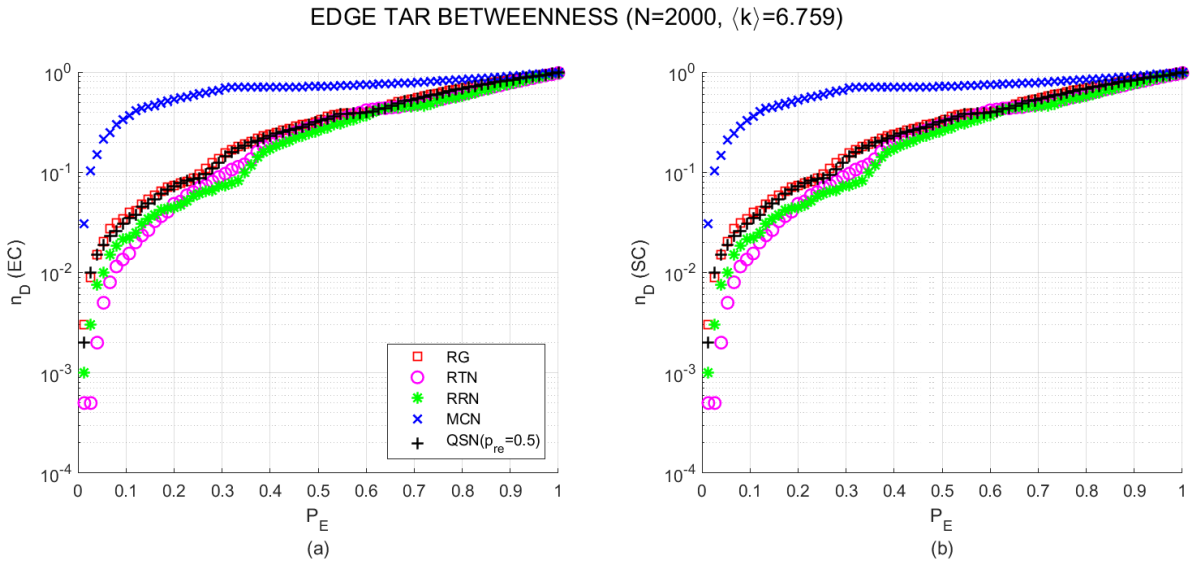
5.3.1 Edge Random Attack

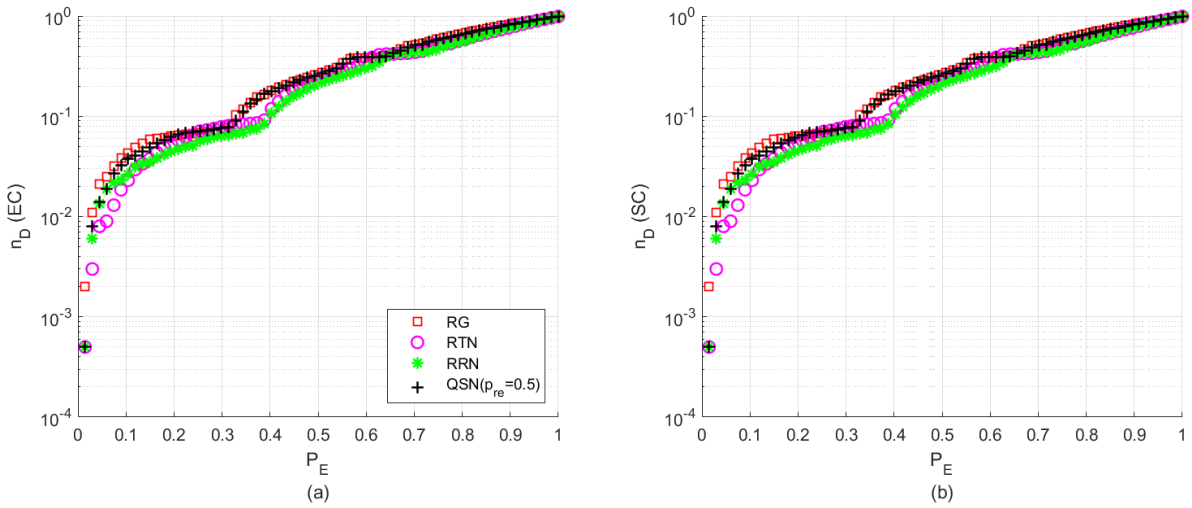


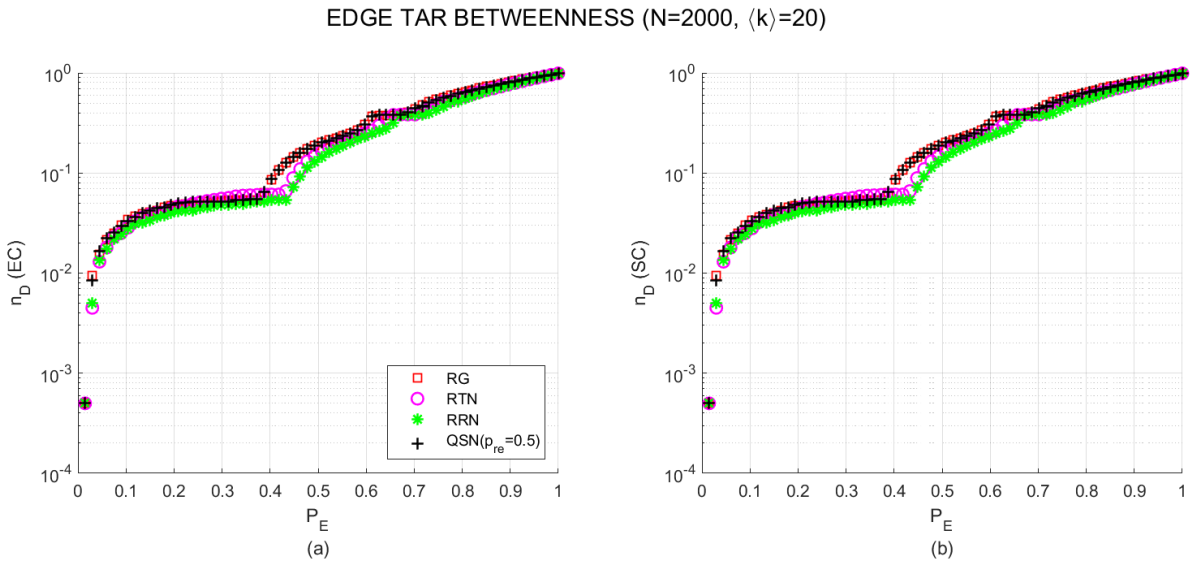



EDGE RAND ATTACK (N=2000, ⟨k⟩=20)
RG
RTN
RRN
QSN(p_re=0.5)
n_D (EC)
P_E
(a)
n_D (SC)
P_E
(b)

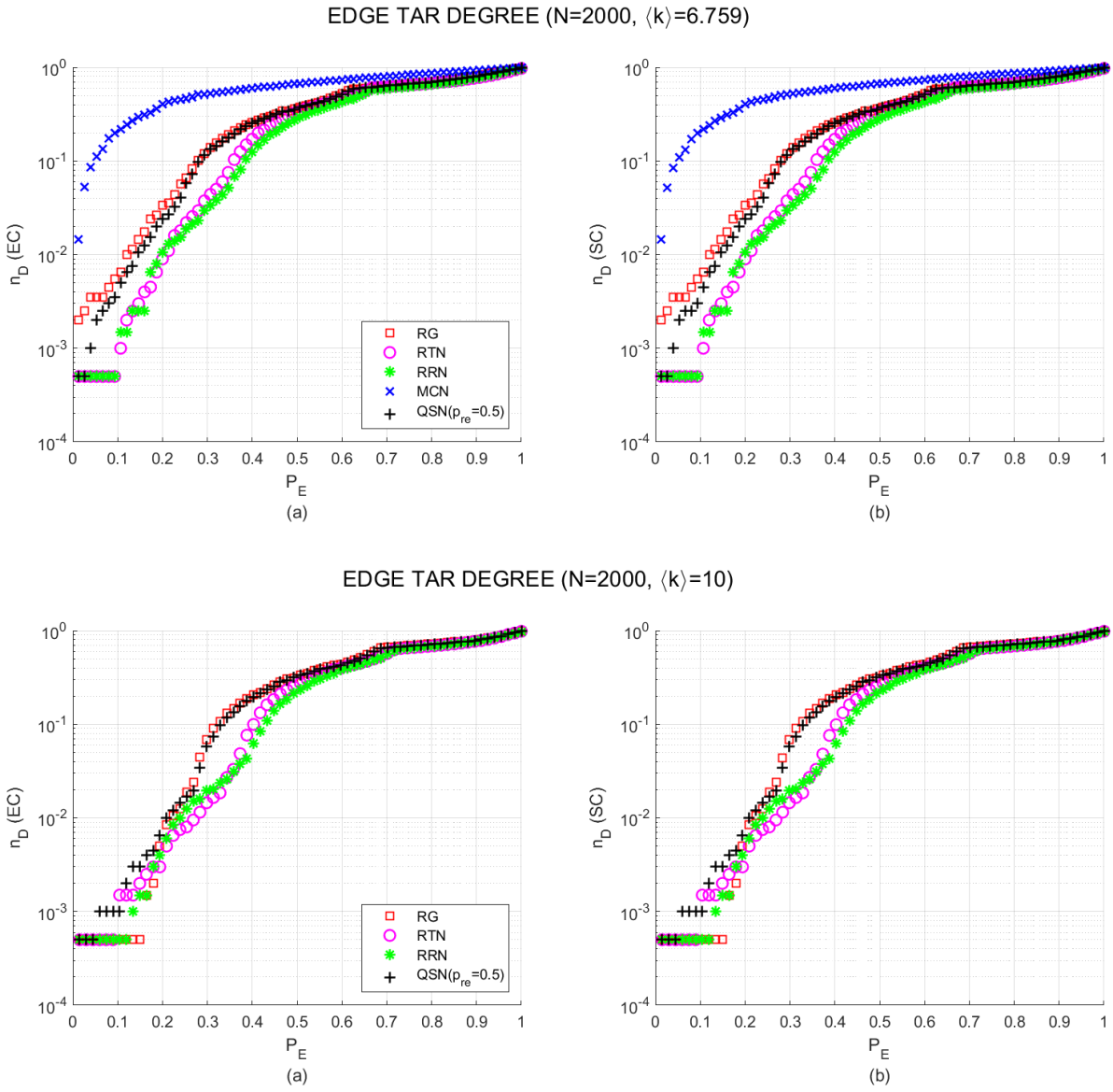
5.3.2 *Edge Intentional (Betweenness-based) Attack*

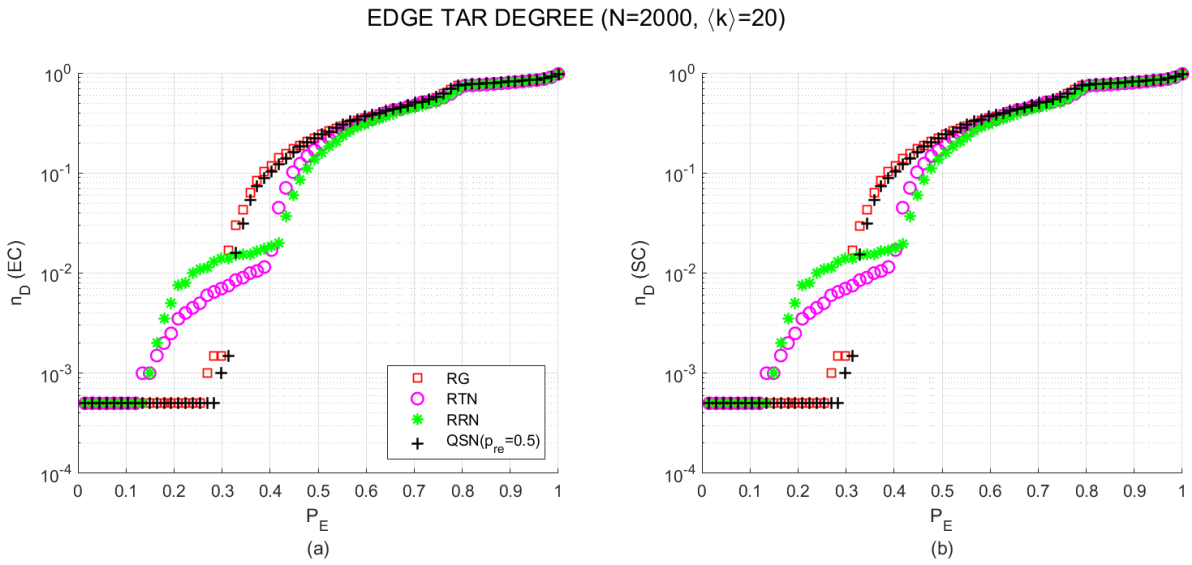




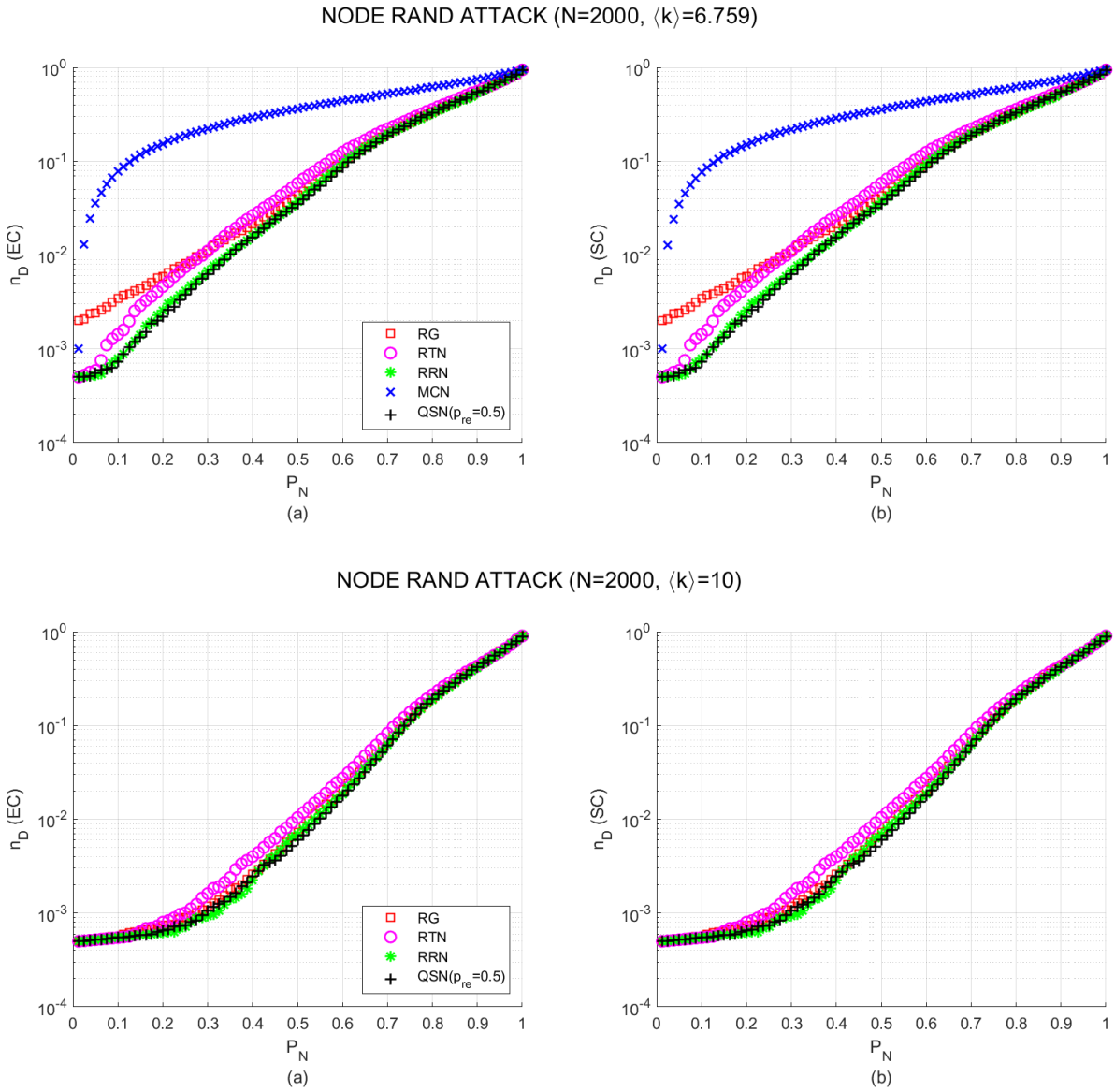

EDGE TAR BETWEENNESS (N=2000, ⟨k⟩=20)
n_D (EC)
P_E
(a)
RG
RTN
RRN
QSN(p_{re}=0.5)
n_D (SC)
P_E
(b)

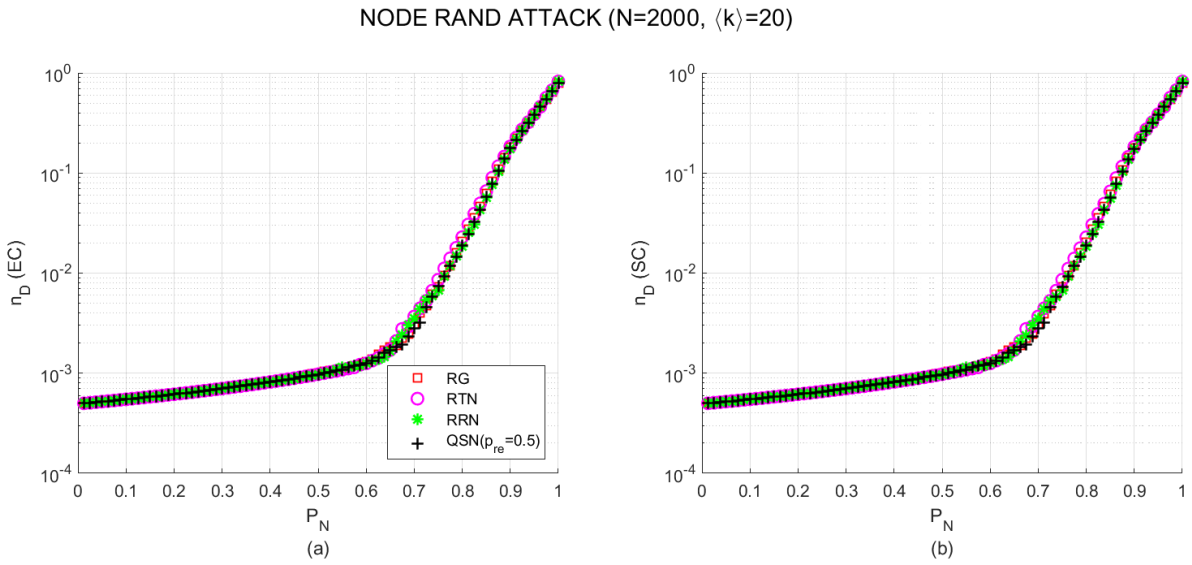
5.3.3 *Edge Intentional (Degree-based) Attack*



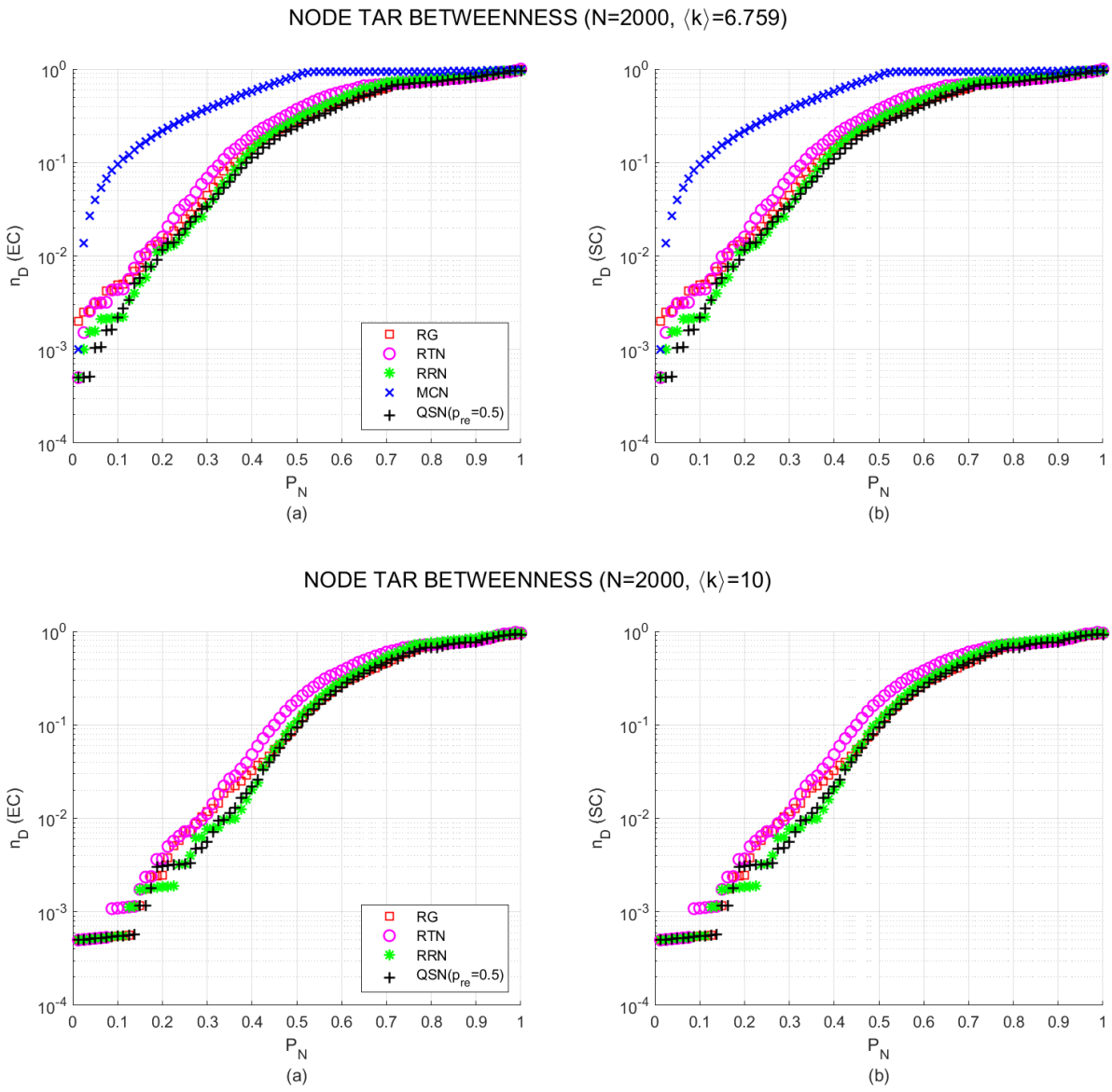

EDGE TAR DEGREE (N=2000, ⟨k⟩=20)
n_D (EC)
P_E
(a)
RG
RTN
RRN
QSN(p_{re}=0.5)
n_D (SC)
P_E
(b)

5.3.4 *Node Random Attack*

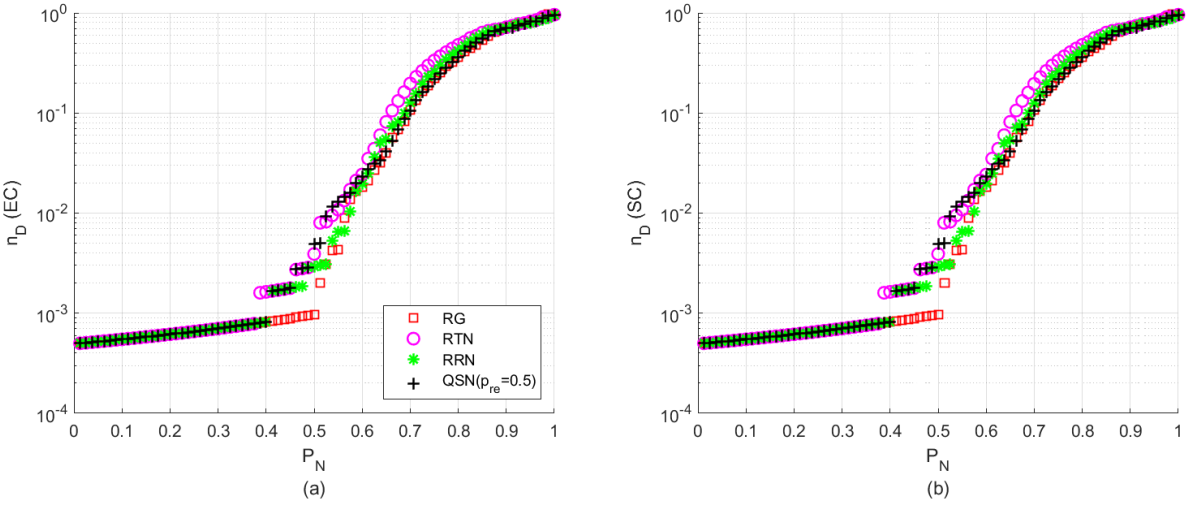



NODE RAND ATTACK (N=2000, ⟨k⟩=20)
n_D (EC)
P_N
(a)
RG
RTN
RRN
QSN(p_{re}=0.5)
n_D (SC)
P_N
(b)

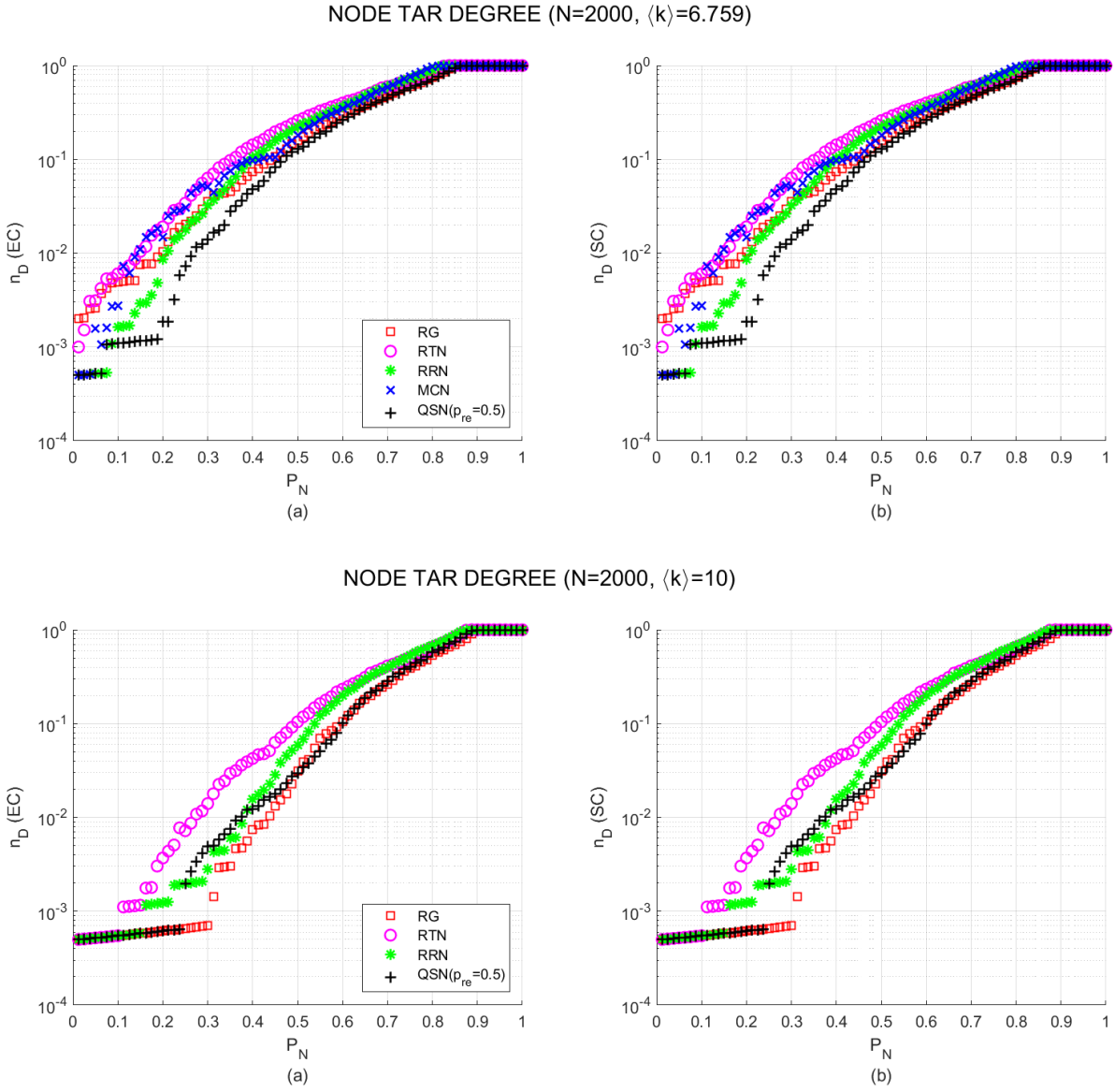
5.3.5 *Node Intentional (Betweenness-based) Attack*

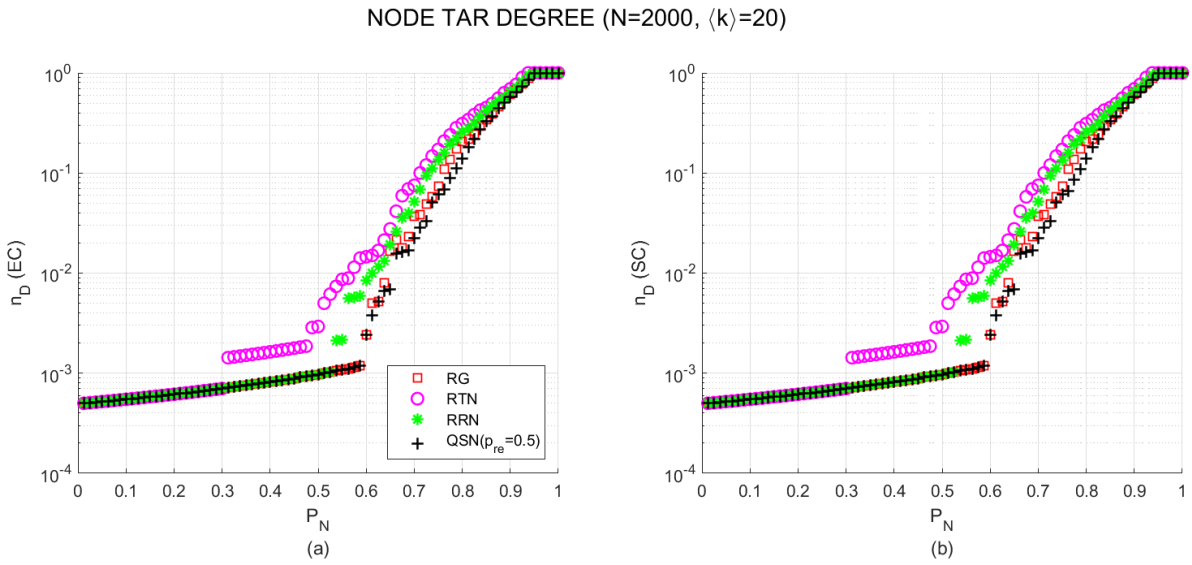


NODE TAR BETWEENNESS (N=2000, ⟨k⟩=20)



5.3.6 *Node Intentional (Degree-based) Attack*




NODE TAR DEGREE (N=2000, ⟨k⟩=20)
n_D (EC)
P_N
(a)
RG
RTN
RRN
QSN(p_{re}=0.5)
n_D (SC)
P_N
(b)